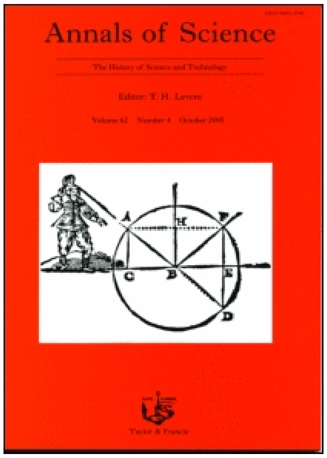

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Descartes's Experimental Journey Past the Prism and Through the Invisible World to the Rainbow

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Descartes's Experimental Journey Past the Prism and Through the Invisible World to the Rainbow

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Summary

Descartes's model for the invisible world has long seemed confined to explanations of known phenomena, with little if anything to offer concerning the empirical investigation of novel processes. Although he did perform experiments, the links between them and the Cartesian model remain difficult to pin down, not least because there are so very few. Indeed, the only account that Descartes ever developed which invokes his model in relation to both quantitative implications and to experiments is the one that he provided for the rainbow. There he described in considerable detail the appearances of colours generated by means of prisms in specific circumstances. We have reproduced these experiments with careful attention to Descartes's requirements. The results provide considerable insight into the otherwise fractured character of his printed discovery narrative. By combining reproduction with attention to the rhetorical structure of Descartes's presentation, we can show that he worked his model in conjunction with experiments to reach a fully quantitative account of the rainbow, including its colours as well as its geometry. In this one instance at least, Descartes produced just the sort of explanatory novelties that the young Newton later did in optics. That Descartes's results in respect to colour are in hindsight specious is of course irrelevant.

Contents

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1. Cartesian Explanation and Experiment

The 'method' published in 1637 by René Descartes (1596–1650) 'for rightly directing one's reason', whose antecedents lay in his unpublished 'rules for the direction of the mind', was accompanied by three discourses: the first on optics, the second on meteorology, and the third on geometry. It is perhaps an interesting coincidence that the eighth discourse in the *Dioptrique*, and the eighth in the *Météores,* are the only two that utilize geometry to produce something that was not known beforehand. The *Dioptrique* uses the law of refraction, which Descartes famously generated in its second discourse, to deduce the anaclastic, the shape of a refracting body that will bring a set of parallel rays to a single point. The *Météores* uses the same law to deduce the angles of the primary and secondary rainbows. There is, however, a marked difference between the two deductions. In the case of the anaclastic, there was no existing natural phenomenon to explain; it was instead a case of generating the specifications for an artificial object that would behave in a certain way. The rainbow on the other hand exists naturally, and its properties accordingly brought issues of observation directly into question. And not only observation, for Descartes's discussion of the rainbow is the only place in all of his publications that he brought together the very different realms of geometry, experiment, and mechanical explanation.[1] Together with Isaac Beeckman (1588–1637), Descartes had conceived in the 1620s of a new way to bring geometry together with mechanics, one that bypassed the Scholastic distinction which subordinated optics or mechanics, when treated mathematically rather than causally, to geometry under the rubric of 'mixed mathematics'. What they and others at the time termed a 'physico-mathematics' sought to bring together the two worlds, so that, John Schuster remarks, 'the old mixed mathematical fields are explained in corpuscular-mechanical terms and therefore are not subordinate to, but are proper domains of, the new natural philosophy'.[2]

Descartes specifically chose the rainbow (Figure 1), 'a wonder of nature', to epitomize his new way to generate knowledge.[3] 'I could not', he wrote, 'choose a

[1] The Cartesian rainbow has been discussed often, notably by Carl B. Boyer, *The Rainbow. From Myth to Mathematics* (New York, 1959), Stephen Gaukroger, *Descartes. An Intellectual Biography* (Oxford, 1995), William R. Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes* (Canton, MA, 1991), A.I. Sabra, *Theories of Light from Descartes to Newton* (Cambridge, 1981), Stephen Gaukroger, *Descartes' System of Natural Philosophy* (Cambridge, 2002), and Richard S. Westfall, 'The Development of Newton's Theory of Color', *Isis* 53 (1962) 339–580, Boyer's now half-century-old discussion remains an insightful introduction not only to Descartes but to the long history of speculations concerning the 'iris' and to the post-Cartesian developments, culminating in George Biddell Airy's nineteenth-century analysis based on interference. Shea and Gaukroger rely primarily on Boyer. A recent book by two optical scientists nicely supplements Boyer (Raymond L. Lee and Alistair B. Fraser, *The Rainbow Bridge. Rainbows in Art, Myth, and Science* (University Park, PA, 2001)). See also M. Minnaert, *The Nature of Light & Color in the Open Air* (New York, 1954), 174–76. Jean-Robert Armogathe, 'The Rainbow: A Privileged Epistemological Model', *Descartes' Natural Philosophy*, edited by Stephen Gaukroger, John Schuster and John Sutton (London, 2000), 249–57, and 'L'arc-En-Ciel Dans Les Météores', *Le Discours Et Sa Méthode*, edited by Nicolas Grimaldi and Jean-Luc Marion (Paris, 1987) 145–62, offers another perspective on Descartes's account.

[2] John Schuster, ''Waterworld': Descartes' Vortical Celestial Mechanics', *The Science of Nature in the Seventeenth Century. Patterns of Change in Early Modern Natural Philosophy*, edited by Peter R. Anstey and John A. Schuster, Studies in History and Philosophy of Science (Dordrecht, 2005), 37.

[3] The original of René Descartes, *Discours De La Méthode Pour Bien Conduire Sa Raison & Chercher La Verité Dans Les Sciences Plus La Dioptrique, Les Méteores, Et La Géometrie, Qui Sont Des Essais De Cette Methode* (Leyden, 1637), with minor orthographic corrections, was published in full in vol. 6 of the first edition of his *Oeuvres*, edited by Charles Adam and Paul Tannery, *Oeuvres De Descartes*, 12 vols. (Paris, 1897–1910) (hereafter AT; *Les Météores* are on pp. 231–366). The whole minus the *Géometrie* was

more appropriate subject for demonstrating how, with the method I am using, we can arrive at knowledge not possessed at all by those whose writings are available to us'.[4] Daniel Garber's illuminating discussion of the methodological structure of Descartes's account nevertheless notes that 'it is by no means obvious how [the rainbow account's] somewhat confused mass of experiment and reasoning can be fit into the rather rigid mold of Descartes's method'.[5] It is well known that Descartes sought to produce a system by means of which particular effects in the world could be generated through paths that, at each step, descended from the more to the less general. It is also well known that his *Discourse* repeatedly invokes the importance of experiment, indeed of a potential infinitude of experiments, in the process. Yet what role experiment was to play in the construction of Descartes's path is difficult to pin down, and indeed remains controversial to this day. Moreover, it is not even clear just what Descartes had in mind when he wrote that the eighth discourse was to provide 'knowledge not possessed at all by those whose writings are available to us'. Where in Descartes's chain of reasoning did previously unheard of knowledge reside?

According to Garber, Cartesian experiment 'is somehow supposed to help us find the right deductions, the ones that pertain to our world and to the phenomena that concern us. In this way, experiments seem not to replace deductions, but to aid us in making the proper deductions'.[6] On this account, a Cartesian experiment is rather a device to weed out alternative deductive paths, which are alone true knowledge, than knowledge in its own right. Any particular path must begin at the bottom, with a phenomenon; the procedure is then to catalogue the set of all relevant immediate factors that might be involved, at which point experiment enters to select among them. The result is then subject to the same procedure, leading backwards up the chain to increasingly general factors. Once completed in this manner, the chain is reversed, thereby reasoning downwards through increasingly specific propositions until the original phenomenon is retrieved. Descartes, Garber remarks, constructed in this way 'the cause of the rainbow', which is 'revealed in the deduction itself'. In that sense the deductive path taken as a whole constitutes Cartesian knowledge properly speaking, and not the switching elements in themselves, namely the experiments, which select this rather than that upward-moving pathway.[7]

Among late sixteenth and early seventeenth century scholastics the production of recondite effects of the sort that Descartes had in mind—those, namely, that did not take place in the course of ordinary experience—was intrinsically problematic just

translated into Latin by the Protestant minister and theologian Etienne de Courcelles (1586–1659) and published in 1644, with Descartes approving the overall sense but not assisting in the translation, which is infelicitous (also included in vol. 6 of AT). 'Of the rainbow' occupies only thirteen and a half pages in translation. The Olscamp translation (René Descartes, *Discourse on Method, Optics, Geometry, and Meteorology*, trans. Paul J. Olscamp, The Library of Liberal Arts (New York, 1965): hereafter PO), where used, has been checked against AT; the corresponding location in AT has been provided in all cases.

[4] PO, 332; AT, v6, 325.

[5] Daniel Garber, 'Descartes and Experiment in the Discourse and Essays', *Essays on the Philosophy and Science of René Descartes*, edited by Stephen Voss (Oxford, 1993), 298.

[6] Garber, 'Descartes and Experiment in the Discourse and Essays', 293–94.

[7] Garber bravely constructs a diagrammatic tree to represent Descartes's account of the rainbow. Though plausible, the path exhibits yawning gaps and requires parallel branchings that inevitably result from the 'confused mass of experiment and reasoning' that Descartes's readers encounter (Garber, 'Descartes and Experiment in the Discourse and Essays', 299).

because it was hard to render such things universal, and proper knowledge was always of the universal, not the particular. Experiments, even arcane productions, certainly were performed during this period, and knowledge was derived from them, but this occurred for the most part among alchemists, who distilled an intricate brew of Aristotelianism and atomism.[8] Yet even François d'Aguilon (1567–1617), a Jesuit whose interests centred on mathematics and the mixed sciences rather than on the broader reaches of natural philosophy, reflected a common view when he wrote (in a treatise on optics) that

> a single act does not greatly aid in the establishment of the science and the settlement of common notions, since error can exist which lies hidden for a single act. But if [the act] is repeated time and again, it strengthens the judgement of truth until finally [that judgement] passes into common assent; whence afterwards [the resulting common notions] are put together, through reasoning, as the first principles of science.[9]

Repetition can transmute the uncertain individuality of arcane and private experience into the gold of 'common assent', which, so transformed, finds its appropriate place within the secure structure of proper demonstration.

For scholastics, then, experiment was primarily a method for converting the particular into the universal, thereby gaining access to the essential form of a body or to the forms of associated effects. Descartes saw it instead as a principle of selection, but this seems scarcely to evolve a truly novel view of experimentation, for so construed Cartesian experiment might appear to be a passive factor in the process of knowledge generation. Yet there is significant originality here, which can best be understood by dividing in three the distinct ways in which Descartes's deductive pathways might be constructed. Two among these three stand apart from one another, while the third deploys both alternatives.

An ideal situation would presumably invoke the Cartesian invisible world throughout and might work in the following way. Start with some particular experimental or observational configuration and ask why the visible property O_1 occurs. Translate the observable configuration into a spectrum of possible microcosmic analogs A_1^i, each of which entails O_1. Now, perform an experiment that yields an observation O_2 which is compatible with only one among these analogs, say A_1^4. To do that requires retranslating each of the A_1^i back into the visible world in such a way that every one of them entails a different result for the same configuration. The next step would be to determine why A_1^4 occurs in this situation. Again, a spectrum of allowable microcosmic arrangements might yield A_1^4, and so the process repeats, moving ever upwards through the deductive branches that delineate the Cartesian microcosm. Here, then, experiment works directly with that world, requiring at every stage a translation between the invisible and the visible.

Alternatively, Descartes's deductive weeding might not need the invisible world at all. It might proceed entirely with observable properties. A given observation might be the result of various possible factors in the visible world; which of these is in fact

[8] On which see William R. Newman, *Atoms and Alchemy. Chymistry & the Experimental Origins of the Scientific Revolution* (Chicago, 2006).

[9] Franciscus Aguilonius, *Opticorum Libri Sex, Philosophis Iuxta Ac Mathematicis Utiles* (Antwerp, 1613), 215–16, cited and translated in Peter Dear, 'The Meanings of Experience', *The Cambridge History of Science. Early Modern Science*, edited by Katherine Park and Lorraine Daston, vol. 3 (2006), 122.

the case might be determined by means of suitable observations without invoking the microcosm. Descartes would still have somehow to envision all of the conceivable, and observable, configurations that would yield the result with which he begins and then to select by means of experiment among them, moving ever upwards to more and more general observations.

In both of these two schemes, the world of possibilities is known beforehand, and it is univocal. The first admits the visible world in order to choose among invisible alternatives; the second eschews the microcosm and uses experiment to select among the spectrum of visible possibilities. Neither seems to offer much scope for experiment as exploration or discovery. There is, however, a third possibility which mixes elements of the two polar alternatives in a way that does involve a directly productive role for discovery. Here, we begin with an observation and then try to manipulate the experimental conditions in order to pin down the factors that will alter the effect we are interested in. This can involve a sequence of experiments and even the modelling of one kind of situation on a different one that, we might argue, is sufficiently similar to the first to provide relevant information. Here, we begin with the visible world, as in the second of our polar alternatives, but we use experiment to discover relationships of dependency that we could not otherwise have known and perhaps not even have envisioned. This procedure will lead us some way up Descartes's branching tree, since it will at least implicitly exclude conceivable paths, but we need not know these paths beforehand.

We might, however, reach a point where we are blocked from further progress in pinning down the observable factor that will produce some effect that engages our attention. We might moreover have observed one or more effects along the way that we considered ancillary to our main goals and that we accordingly set to the side. It is precisely here that the methods of our microcosmic first alternative may come to our assistance. We may translate the observed configuration into its invisible analogue and ask ourselves whether the information that we had previously set to the side might now convey something useful about interactions among the elements of the microcosm. We may be able to construct several possible alternatives among which the discarded observations will select just one. And that, when translated back into the visible realm, may provide a route that permits further progress.

An essential part of Descartes's procedure in the case of the rainbow involved precisely this working interplay between his model of the invisible world and his experimental procedures. I shall argue in what follows that Descartes's explanation of the very *existence of colours* in the rainbow emerged out of a productive engagement with his invisible world on the basis of an effect that he did not know before he experimented with a prism. I shall further argue that he was able to fabricate an explanation of the *order of colours* in both the primary and secondary bows by extending to them a property which he also discovered by means of prismatic experiments. That the property in question is in hindsight specious is of course irrelevant.

The argument that follows grants meaning to what seem to be ruptures in Descartes's text, and it is therefore inferential. We will see that Descartes's abrupt excursion into the invisible world is prompted by the difference in colours at either end of a light patch produced by the prism, but that the connection between the prismatic configuration and the coloration is not actually made as he wanders through the microcosm. At one point he emphasizes an apparent exception to coloration that he loosely connects to the invisible realm only to detour from that into an excursis on the reason for why colours might appear at all in the case of the rainbow. Later, after considering the paths of rays through a raindrop, he turns abruptly back to the prism to remark something that seems to have nothing to do with colour, and which he explains obscurely, only to attempt an equally obscure connection to the raindrop in order to explain the colour inversion between the primary and secondary bows. Each of these apparent ruptures and obscurities, I will argue, reflects the difficulties that naturally arose as Descartes composed his narrative, as he sought to reason through the thicket of possible connections that his prismatic experiments afforded, and to link them to the raindrop in the light of the several differences between prism and drop.

To offer further support for my claims, I reproduced Descartes's experiments w ith a water-filled glass bowl and with a prism. As a result, what might appear in the absence of experimental reproduction to be two factors that could easily be overlooked or underestimated on reading Descartes's text take on a compelling significance: namely, the width of the aperture through which Descartes passed the prism's light, and the appearance in the case of a sufficiently small opening of an anomalous tint.

Descartes's narrative will accordingly be read in a new way. Instead of seeing it exclusively as a rather chaotic attempt to apply 'method', I will instead read it as a complex and incompletely expressed account of the most intricate effort that Descartes ever made to link together the very different worlds of experiment, geometry, and mechanical reasoning, an attempt to forge not merely a result, but the very procedures for making a unity out of a disparate triad. Written to persuade, Descartes's tale was designed to carry the reader along on a seemingly inevitable journey to an unavoidable conclusion. Not surprisingly, this Cartesian tale twists and turns upon itself, mixing and matching its several elements in ways that were nearly guaranteed to puzzle his readers, as it certainly did at the time and has done ever since.

2. Experiments with a Water-Filled Sphere

I begin with the essential point around which Descartes framed the narrative, for it was hardly new. He had certainly read as much in Franciscus Maurolycus (1494–1575)[10]; moreover, he had also at least perused the synoptic account of earlier work by Libert Froidmont (1587–1653),[11] who had discussed its several versions: namely, that the rainbow is formed by sunlight that has been reflected internally by raindrops. Descartes affirmed the claim by noting that rainbows can also appear in fountains, and he further noted that he had previously argued, elsewhere in the *Météores*, that these drops are spheres. He had been anticipated in his next step, to model the raindrop by means of a spherical water-filled flask,[12] by Theodoric of Freiburg as far back as the thirteenth century. He might also have read a claim by

[10] Franciscus Maurolycus, *Abbatis Francisci Mavrolyci Messanensis Photismi De Lvmine, & Vmbra Ad Perspectiuam, & Radiorum Incidentiam Facientes* ... (Naples, 1611).

[11] Libert Froidmont, *Liberti Fromondi Meteorologicorum* (Antwerp, 1627).

[12] Under the assumption that size does not make the colours and their disposition appear 'in any other way' than they would in the natural phenomenon: AT, v6, 325.

Marco Antonio de Dominis (1564–1624) that he had done something like this,[13] though the usefulness of modelling by means of a water-filled globe could have been suggested by Descartes's claimed study of Kepler's *Astronomia pars optica*,[14] with the latter's extensive discussion of refraction through an 'aqueous sphere'. In any case, Froidmont mentions that an 'artificial' rainbow can be produced using several means, including a urinal or a wine flask.[15]

One drop does not a rainbow make; it produces instead images of the Sun at various angles, with the rainbow resulting from a spray of drops at different heights, each sending to the eye light that has struck it at a particular angle of incidence, forming a 'rainbow mosaic'.[16] Descartes's aim was to see whether the images would reproduce in colour and position the characteristics of the rainbow. He described looking at either the bottom or at the top of the flask, raising or lowering it to observe how the light changed. The experiment can be reproduced quite simply with a spherical glass bowl. Set the flask into a position such that the Sun's image seen through its bottom shines as brightly as possible, and the image will be quite red. A slight motion downwards turns the red a slightly dimmer yellow; continue down and the image turns green, then blue, and finally becomes too dim to see. Starting back at the red, a slight motion upwards will cut out the light altogether.

Figure 2 is a photograph of the red-to-yellow transition for the image. The observation is not simple to make with a high degree of accuracy because the transitions occur quickly, and because the Sun is hardly a point source. To measure approximately the angle from the Sun to the point of the flask where its image emerges and then to the eye—which I will call the viewing angle[17]—in the simplest way requires either a sextant-like instrument or else just marked sticks to take the solar altitude by its shadow, the height of the point where the Sun's image emerges, as well as the height of the observing eye and its horizontal distance to the point of emergence. Descartes did not specify any procedure at all for taking his measurements, but he found that the viewing angle is 'approximately 42°'. He further

[13] In the *Opticks*, Newton implied that Descartes drew nearly everything worthwhile from de Dominis, who taught 'how the interior Bow is made in round Drops of Rain by two refractions of the Sun's Light, and one reflexion between them, and the exterior by two refractions and two sorts of reflexions between them in Each Drop of Water'. Further, Newton wrote that de Dominis 'proves his Explications by Experiments made with a Phial full of water, and with Globes of Glass filled with Water, and placed in the Sun to make the Colours of the two Bows appear in them. The same Explication Des Cartes hath pursued in his Meteors' (Isaac Newton, *Opticks: Or, a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light. Also Two Treatises of the Species and Magnitude of Curvilinear Figures* (London: Sam Smith and Benjamin Walford, Printers to the Royal Society, 1704), 127). Although the De Dominis book was in his library, Newton's characterization of the explication is wide of the mark, since de Dominis never mentions an emergent refraction for either the primary or the secondary bow, or two reflections for the secondary: see Marci Antonii de Dominis, *De Radiis Visus Et Lucis in Vitris Perspectivis Et Iride* (Venetiis, 1611); de Dominis' account is discussed in Boyer, *The Rainbow. From Myth to Mathematics*, 187–92, R.E. Ockenden, 'Marco Antonio De Dominis and His Explanation of the Rainbow', *Isis* 26 (1936), and on Newton see Alan E. Shapiro, ed., *The Optical Papers of Isaac Newton. The Optical Lectures. 1670–1672*, vol. 1 (Cambridge, 1984), 593, note 1.

[14] AT, v6, 325.

[15] Froidmont, *Liberti Fromondi Meteorologicorum*, 358, noted in Armogathe, 'The Rainbow: A Privileged Epistemological Model', 252. Froidmont remarks 'Sed alterum Iridis artificialis genus est, merae διακλασιοζ refractionis filia. Talem prismata, & vitra triangularia efficiunt. Item urinale, aut vitru etiam vulgare vinarium . . . '. That is, an artificial kind of rainbow can be produced with a 'triangular' glass (i.e. a prism), a urinal, or a wine flask.

[16] In the felicitous phrase of Lee and Fraser, *The Rainbow Bridge. Rainbows in Art, Myth, and Science.*

[17] This is, in modern parlance, the scattering angle for a refracting sphere.

observed that the solar image also appears towards the top of his flask, where it again shines most brightly with the colour red. Here, however, moving the flask downwards cuts out the light, while moving it upwards passes to yellow and so on. In addition, the image at this upper region is much less bright than at the lower, and the corresponding viewing angle is 'around 52°'.[18]

Descartes did not specify the size of his flask nor how far out he positioned it, but if a 15-cm flask is placed at arm's length and observed at such an angle that the top is at peak shine, then the shine will also peak at the bottom. This is about a 10° angular difference between top and bottom, which is also close to the angle between the outer arc of the primary and the inner arc of the secondary rainbow. In other words, the visual angle subtended by a 15-cm flask held at arm's length nicely fits the situation that Descartes was looking for, and he may well have chosen the flask size accordingly. Or he may just have used whatever reasonably sized flask—probably, as in Froidmont's suggestion, a urinal—was available to him. To make measurements, he would have had to fix the flask in place, in which case he could have chosen any suitable distance from which to observe it.

'After this'—after these basic observations—Descartes examined 'in more detail what caused' the lower image 'to appear red'.[19] He 'found' that the image resulted from the refraction of a ray that strikes the upper part of the drop, followed by an internal reflection and then a second refraction at emergence. This is not hard to discover, given the basic idea that one or more internal reflections are involved, though it requires a bit of manipulation. First of all, that the (lower) image comes from a top-entering ray can be found simply by blocking various points of the surface. To find that it then emerges after a single internal reflection is more difficult, but it can be done by placing a stick into the flask and manoeuvring it until the image cuts out, at which point one can see the spot of the Sun on the stick, marking where it would otherwise strike the inner surface. The upper image poses more of a problem. Though it is again simple to determine that it is due to a ray that strikes near the bottom of the flask, it is a little harder to see that it involves two internal reflections. To do so requires manoeuvring the stick first to block one of the reflections and then shifting it to block the other. Descartes explicitly stated that he blocked light to find the loci of internal reflection.

In his published account, Descartes insisted that the specifics of the rainbow's geometry, in particular the angles at which the light cuts out, could not be determined solely by observation, but that a proper understanding of the processes involved was essential. To underpin the point, he cited the inaccuracy of Maurolycus' angles. One might consider Descartes's remark a rhetorical ploy designed to push the priority of geometrical and causal knowledge over the results of untutored observation or experiment. And it is certainly at least that. But it is also considerably more, because it reflects Descartes's own experience with his water-filled flask. The reproduction shows that it is simply not possible with this sort of device to pinpoint the cutoff angle to better than a degree or two, and even that is extremely difficult. The image is first of all not a point but a patch (Figure 2) because of the Sun's finite angular size. It is moreover highly coloured, with quick changes in coloration as the flask is raised or lowered. To locate the point at which the light cuts out altogether

[18] AT, v6, 326–27.
[19] PO, 334: AT, v6, 328.

Figure 1. Primary and secondary bows just before sunset (photo © D. Bush, by permission).

Figure 2. Singly reflected solar image at the transition from red to yellow within a spherical, water-filled flask. The unreflected, emergent ray is visible on the metal column from which the flask hangs.

with any degree of precision, the Sunlight would have to be passed through a lens so that all rays come from the focal point, thereby avoiding the problem of solar width. But coloration would still pose a problem for visual pinpointing of the cutoff angle. As a result, it is reasonably certain that Descartes filtered the demands of rhetorical persuasion through his actual experience with the flask, in which case the nitty-gritty of experimental practice likely entered at the earliest stages of Descartes's work with the rainbow.

3. Using a Prism to Probe the Conditions of Colour Production and of Colour Order

Thus far, Descartes's narrative is quite straightforward and, my reproduction indicates, probably does reflect what he had actually done. Having obtained the essential characteristics of the phenomenon, Descartes turned to the 'principal difficulty', as he called it, and it is at this point that the tale begins to twist.[20] At any given height of the flask, with the eye looking always at the same point on the bottom or the top, there is always some incident ray that will reach the eye after, respectively, one or two internal reflections. Why is it, then, that 'nonetheless only those of which I have spoken cause certain colours to appear'?[21] Note that Descartes has framed the question in immediate conjunction with colour: why are 'certain' colours appearing under these circumstances and not under others?

There are two questions here.[22] One concerns the image's brightness, which peaks at the two select angles, cutting out above the lower and below the upper, and tailing off more slowly in the opposite directions. The other concerns colour proper: red appears at the most 'brilliant' loci, with 'yellow, blue, and other colors' in the tail-off regions.[23] These tail-off regions are mirrored symmetrically at the upper and lower angles in that the red is at the least angle for the upper position and at the greatest angle for the lower, with succeeding colours following in the same order at both, albeit with opposite relations to the succession of viewing angles. According to his published account, and we have no other, Descartes turned first to the question of colour, implicitly separating it from that of brightness. The separation must have occurred quite early in the discovery process, enabling Descartes to concentrate directly on the colours proper. We will see below how he brought the two together again. The problem of colours, at this point, itself became twofold: first, what circumstances are necessary to produce colours at all in situations like this one; second, what are the conditions that determine the order of colours, and are these conditions invariant? But these two issues were not clearly distinguished from one another, and the question of colour order posed particularly difficult problems that are in good measure responsible for the fractured character of Descartes's narrative past his discussion of the flask observations.

It is upon the presence of 'colours' at certain points only—not just the presence there of light proper—that Descartes concentrated, and 'to resolve this difficulty', he continued, 'I looked to see if there were some other subject where [the colours] appeared in the same way, so that by comparing them with each other I could better judge their cause'. He decided that this could best be done by examining the colours produced by refraction through a prism under suitably constrained circumstances. The prism became, as it were, a restricted model of his water-filled flask, which was itself a model of the raindrop—a twofold displacement. The modelling prism then had to produce colours 'in the same way' ['en même sorte'] as the flask, so Descartes had to work out just what 'the same way' meant. And finally there is his goal—to 'better judge their [the colours'] cause'. It is precisely here that the narrative becomes especially difficult, because Descartes was trying to forge a logical path out of the complex mix that he had brewed while working with the prism.

Cause, here, has two distinct but related meanings: there are the experimental conditions which are necessary to produce the colours, and then there are the

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[20] PO, 334; AT, v6, 329.
[21] Ibid.
[22] Garber, 'Descartes and Experiment in the Discourse and Essays', 298 notes the point.
[23] PO, 333; AT, v6, 327.

presumptive mechanical properties of light itself which, under these conditions, are activated in order eventually to engender the sensation of colour. These are two different senses of 'cause', since the former—the experimental conditions—might well hold whether or not the mechanical structure remains acceptable. On the other hand, the mechanism must be activated whenever the conditions hold. Descartes's account seeks to link the two together, but in doing so, it seems to raise side observations whose importance is unclear while turning rapidly to the mechanism, whose initial connection with the details of the prism observations is itself obscure. These narrative fractures have made it difficult for at least some of his readers at the time, as well as for many historians, to appreciate the full extent of his argument, and with good reason, for they most probably reflect the entwined complexity of his work as he tried to tame prismatic behaviour by subjecting it to the demands of mechanism. I will try to re-establish the original agonistic field—the tension between prism and mechanism—in order to make sense of Descartes's full explanation of the order of colours here, and *ipso facto* in the rainbow as well.

In working with the prism, then, Descartes focused on the *conditions of colour production* and the *specification of colour order*, which we can see exemplified in the very configuration of his prismatic experiment. His Figure 3 depicts a right-angled prism to which he attached a 'cloth or white paper' to extend past the prism's edge PM while remaining parallel to it, onto which the light from the Sun was cast.[24] Descartes's configuration confines attention entirely to the production of colours by a single refraction in conjunction with a beam-creating aperture. The design places prism, aperture, and screen in direct proximity to one another, thereby excluding from consideration, or minimizing, other factors, such as the spread of the beam as it leaves the prism.

Descartes did not specify the size of his prism or its index of refraction, but a configuration of this sort places severe limits on what can be observed, not least because of internal reflection. Judging from his remarks, Descartes may have had two prisms, with respective angles $\angle PNM$ of 30° or 40°.[25] If the index were, for example, a not unreasonable (for glass) 1.5, and $\angle PNM$ were 40°, then a ray which strikes the surface NM counter-clockwise from the perpendicular will not emerge from the bottom surface NP unless its incidence is less than about 3°, whereas a ray hitting clockwise will emerge whatever the incidence may be.[26] A 30° prism permits counter-clockwise rays to exit up to an incidence of about 18°. These rays, when they exist, emerge at greater angles than equally incident clockwise rays.

Descartes must early have known, or perhaps have discovered, that he needed an aperture to generate colours by means of prisms in order to produce boundaries between lit and unlit regions.[27] The notion that colours involve the interaction of light with dark was as old as Aristotle, and probably older, and had been discussed by Froidmont. Within the mechanical philosophy, which rejected hylomorphism, the Aristotelian conception's reliance on formal change to explain the interaction made little sense, and to be retained at all, the light–dark dichotomy had to be translated

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[24] PO, 335; AT, v6, 330.

[25] Although we do not know whether Descartes closely controlled the dimensions of figure 3 as printed, measurement of the figure itself yields angles $\angle PNM$, $\angle NMP$, respectively, of 38° 40′, 51° 20′.

[26] For an index n of refraction, total internal reflection sets in for counter-clockwise rays when their angle of incidence on NM reaches $\sin^{-1}\{n\sin[\sin^{-1}(1/n) - \angle PNM]\}$.

[27] See immediately below for a discussion of what seems at first to be a strange claim in respect to apertures.

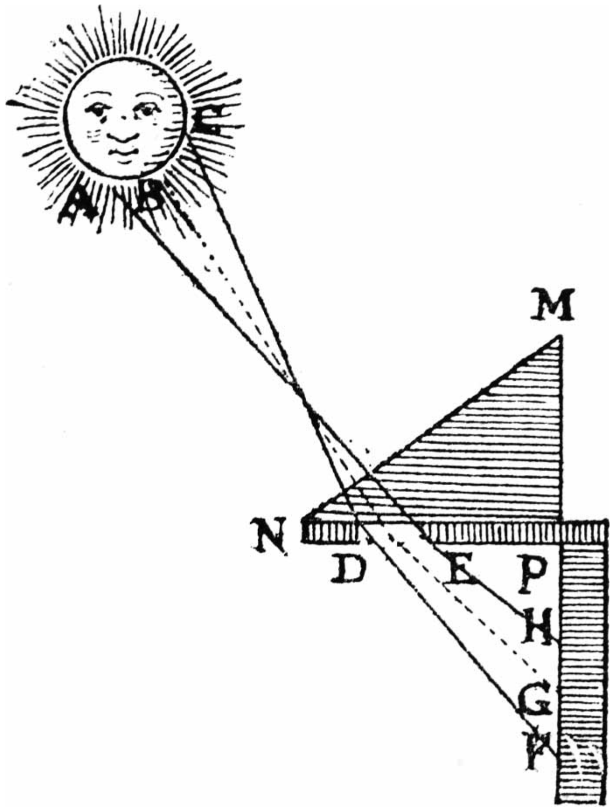

Figure 3. Descartes's colour-generating prism, exit-aperture, and attached screen.

into mechanical effects in the boundary region. Descartes in fact jettisoned the general requirement that only the interaction of light with shadow can produce colours, but he retained a transformed version of it for prismatic tints.[28] This permitted him to use shade boundaries as a critical element in the production of the sorts of colours that scholastics had termed 'emphatic' in order to distinguish them from the colours of bodies, which inhered in them formally.[29] Immediately after describing the apparatus, he remarked:

[28] The placing of colours as intermediates on a chromatic scale ranging from white to black had been abandoned by artists for pigment mixing by the end of the sixteenth century, many of whom considered neither white nor black to be colours or generators of colours. Their views had become increasingly common by mid-century, on which see Alan E. Shapiro, 'Artists' Colors and Newton's Colors', *Isis* 85 (1994), 627, who remarks that the 'widespread use of color mixing culminated in the early seventeenth century in the disclosure of the painters' primaries, and henceforth the painters' trinity would play a fundamental role in color theory ... A transformation had also occurred in the very conception of what a color is, with the shift from a tonal classification based on black and white to a chromatic one based on hue and a separate black–gray–white scale'. Descartes did not apparently adopt the latter scheme, because, we shall see below, he used a mechanical structure to locate white between red and blue, though black was necessarily the simple absence of light.

[29] For Descartes, bodily and emphatic colours could not be different in kind: both were produced in the final instance by the actions of the very same mechanical configurations on the visual apparatus. The configurations could, however, be generated originally in alternative ways, so that the difference between the bodily and the emphatic now became one between their originating mechanical causes and not one of form.

> When I covered one of these two surfaces [viz. *NM* or *NP* in Figure 3] with a dark body, in which there was a rather narrow opening such as *DE*, I observed that the rays, passing through this opening and from there going to contact the cloth or whiter paper *FGH*, paint all the colours of the rainbow there, and that they always paint the colour red at *F*, and the colour blue or violet at *H*.[30]

The need to pass light through an aperture likely determined Descartes's requirement that the Sun's rays should strike the prism as close to the perpendicular as possible, for he wanted to restrict the active refraction to a single surface—the prism's bottom (*NP*)—in order to control the circumstances under which colours are generated. Two refracting surfaces would complicate the factors in ways that would be difficult to disentangle. He could then work with the other factor involved in colour production, the aperture, by placing it on the prism's bottom and then altering its width to vary the image's size, thereby showing that the spread of colours decreases with image size in his experimental configuration. This was, to my knowledge, a new observation, though perhaps one to be expected on the grounds that light must interact with dark to generate colour—since, presumably, the smaller the extent of the illuminated region, the more it might be said to be affected by neighbouring dark.

Descartes's insistence on the role of the aperture might seem odd, since anyone who has played with a prism knows that it is entirely possible to generate a good spectrum by casting the Sun's light on a wall or the ground without blocking any part of the prism face. However, to do so the surface that receives the light should be a fair distance from the prism. If it is not, if the light is caught too close to the prism, then the beam will be essentially white throughout but tinted red at one edge, blue at the other. From Descartes's point of view, an uncovered prism would be thought of as having an aperture the width of the face at which the light enters it. Colours then appear only at the beam termini when the beam is cast close to the prism, but throughout it as well when the beam is cast farther away. As the distance from the region of emergence increases, the ratio of aperture width to distance decreases, and this is for Descartes what determines the extent to which the beam shows coloration—the smaller the ratio, the more thoroughly and variedly tinted the beam. In his experiments, Descartes wished to control the colour-producing factor, and so he held fixed the prism-to-screen distance while varying the aperture size until it was small enough to produce fully tinted light on the screen.[31]

Figure 4 (left) is an image produced on a white-paper screen placed using Descartes's experimental arrangement, with the prism bottom covered and an aperture produced by poking a sharp pencil through the cover, which consisted of black paper. The prism was held so that sunlight struck it nearly orthogonally as he required. The image is vividly coloured, showing red and yellow near the bottom (*F* in Descartes's Figure 3), and greenish blue shading off into violet at the top (*H* in Descartes's figure)—what Descartes described, later in his narrative, as a

[30] PO, 335; AT, v6, 330.

[31] In terms of Descartes's mechanics, which associates increased perturbation of the normal state (white) inversely with aperture size, even a wide aperture will produce coloration, but its mechanical effect on the microspheres (see below) will not become visible except at the edges until the beam has sufficiently diverged—before that the microspheres interfere with one another, leaving the overall state (white) visibly unchanged.

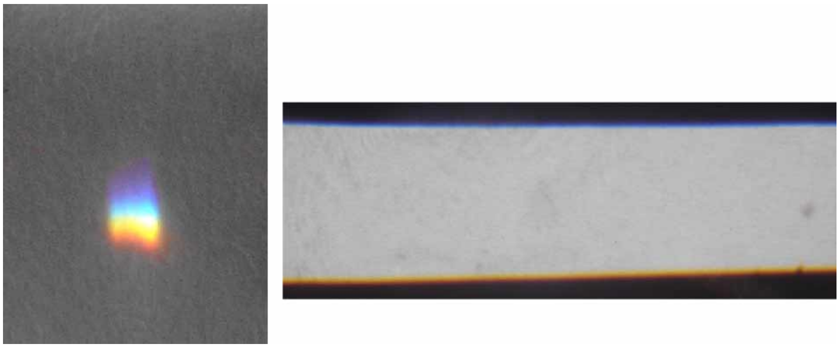

Figure 4. Solar images produced by Descartes's prism with a very small hole (left), and with a 4-mm aperture (right).

'blue . . . mixed with a rosy colour at its edges'—with a small white space separating the coloured regions.

The coloration does indeed change markedly as the aperture size is increased. If the aperture is even a few millimetres wide, the intervening white space occupies most of the image, with the red–yellow and blue–violet receding to thin borders at the top and bottom. A millimetre or two more, and the coloration becomes a thin line at the aperture's boundaries, visibly reduced to a less vibrant red with a slight hint of yellow and to a nearly pure blue, as we can see from Figure 4 right where the aperture was about 4 mm on a prism whose dimensions were 38 mm and 55 mm, respectively, for *NP* and *PM*.[32] 'If we remove the dark body on *NP*', Descartes wrote:

> the colors *FGH* cease to appear; and if we make the opening *DE* large enough, the red, orange and yellow at *F* extend no further because of that than do the green, blue and violet at *H*—instead, all the extra space between the two at *G* remains white.[33]

Although this was the only specific remark that Descartes made about aperture size, I will argue in what follows that his discovery of coloration's inverse relation to it was critical in leading him to an understanding of how colours are produced in the case of the rainbow, where no aperture at all is present. Reproducing Descartes's experiment shows just how striking the effect is, emphasizing as it does the rapid retreat of colours to the boundaries of the image as the aperture grows even slightly. The overall terseness of Descartes's account, together with its evident, if fractured, attempt to order observations and explanations into a coherent series, certainly does obscure the place of this and other thoughts and observations in the original course of his work. Nevertheless, and despite his general mastery of narrative structure, I will argue that Descartes was not able thoroughly to erase the paths that eventuated in his printed account.

The shape of his prism next enabled Descartes, in an abrupt and consequential textual shift, to specify a circumstance that was *not* necessary for colour production:

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[32] This right-angled prism is isosceles, and so its $\angle PNM$ exceeded Descartes's by 5°.

[33] PO, 336; AT, v6, 331.

since the prism has flat sides, curvature is not required to produce coloration—an obvious forward reference to the rainbow, indicating that the curvature of a raindrop cannot be involved in colour generation. Another condition that need not be satisfied, he continued without any explanation as to its significance, is 'the angle under which they [the colours] appear', by which he meant the light's angle of emergence from *NP*. This leads him to a particularly important remark concerning coloration conditions. It is possible, Descartes wrote, to 'cause the rays going toward *F* to curve sometimes more or sometimes less than those going toward *H* ...'.[34]

The 'curve' of an emergent ray towards *F* or towards *H* means its angle with respect to the normal at emergence or, equivalently, that angle's complement, namely with respect to the bottom *NP* of the prism. In Descartes's figure, the ray *EH*, which derives from the counterclockwise incident ray from *A*, emerges at a greater angle than the ray *DF* which derives from the clockwise-incident ray from *C*, and so $\angle PHE$ is greater than $\angle PFD$. This will certainly be the case if the prism is oriented so that the rays which strike the edges of the aperture derive from rays that are oppositely incident on *NM* at equally small angles, as in his figure. If the prism is instead tilted in such a way that the rays from *C* and *A* are both clockwise-incident, then the order of refraction does indeed reverse.

The significance of the altered orientation lies in Descartes's further observation that, even under these circumstances, the rays to *F* 'nevertheless always paint red, and those going toward *H* always paint blue'.[35] The order of colours, in other words, does not depend in any direct way on the order of refractions.[36] It must, however, depend on something that remains the same between the two situations. At this stage in his narrative Descartes chose to keep silent about what this might be, and his confusing quiet here may be a linguistic reflection of the difficulties that he had experienced in reaching a solution.

The essential requirements for producing colour were, then, threefold: the light must be bounded by an unlit region or regions, the region must be narrow in order for the colours to spread throughout the image, and the light must undergo 'at least one refraction, and even one such that its effect was not destroyed by another'—note that two refractions always occur in producing the rainbow.[37] These necessary conditions were embedded among others that might be thought relevant but that

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34 PO, 335; AT, v6, 330.

35 Ibid.

36 In Descartes's experimental configuration, the beam within the prism is effectively undispersed. Furthermore, since he used a right-angle prism with the sunlight entering its long face *NM*, the beam will always emerge from the surface *NP* refracted towards the other side of the prism, *MP*. Because blue light has a higher index than red, the border colour at the edge *EH* must then show blue, and conversely, red always shows at the border *DF*. Where, for Newton, the boundary coloration would exemplify the unequal refrangibility of red and blue light, for Descartes it meant that the order of colours could not be linked to the relative angles of emergence of the beam's edges.

Confusion about what Descartes had in mind with respect to the prism's colour orders remains to this day. Lee and Fraser, *The Rainbow Bridge. Rainbows in Art, Myth, and Science*, 355, note 195, for example, look to Descartes's diagram to conclude that 'he must mean that the red ray emerges from the prism at a less oblique angle than the blue, as his accompanying diagram shows'. On the other hand Sabra, *Theories of Light from Descartes to Newton*, 64 correctly notes Descartes's claim.

37 The original reads 'Mais j'ay jugé qu'il y en falloit pour le moins une, & mesme une don't l'effect ne fust point destruit par une contraire': AT, v6, 330. We will return below to the question of what Descartes meant by a refraction whose 'effect' is not destroyed by a subsequent one, particularly since, in the rainbow, the second refraction produces a ray that emerges at the same angle with respect to the drop normal at which the incoming ray had entered.

prove not to be, including the one just discussed. These, as it were exclusionary, conditions divide into two kinds: those which assert that the very *existence of colours* does not require more than one refraction or curved surfaces, and those which assert that the *order of colours* depends neither on the size of the aperture nor on the order of refractions.

4. A Journey Through the Invisible World

Descartes's mechanism makes its first appearance in the midst of this mélange of inclusive and exclusive conditions. And then, after an extraordinarily challenging voyage through his imperceptible world, Descartes turned to the geometric location of light, followed immediately by a return to the question of colour order, after which he did not bring back the mechanism. Why did he bring in the microworld at this point in his narrative, after elaborating his list of requirements? Consider his words: 'after this', Descartes wrote following the list, 'I tried to understand why these colours are different at *H* and *F*, even though the refraction, shadow, and light concur there in the same way'.[38] The narrative turns immediately to mechanism, so that the purpose of the journey would seem to be to find a cause, not primarily for the existence per se of colours under these conditions, but for the difference between the colours at *F* and *H*.

There were two sorts of cause to which Descartes could have turned, namely to the experimental conditions proper, or to the one that his narrative does evoke, namely the hidden mechanical world which should imply those conditions. To see what Descartes might have done, let us suppose for a moment that, *contra natura*, prismatic refraction does not produce red and blue termini with intermediate coloration. Suppose instead that it produces, say, a red cap at both termini with white in between. If that had been so, would Descartes have turned at this point to the imperceptible world? It seems improbable, because like circumstances ('the refraction, shadow, and light concur there in the same way') would have been producing like effects. And if there is no difference why hunt for an explanation, at least here, where Descartes was trying to generate 'knowledge not possessed at all by those whose writings are available to us'[39]—and by knowledge he seems at least to have meant observable properties?

The ends of the image do, however, show different colours, even though conditions seem to be the same—or, at least, Descartes had not to this point been able to discern what the change might be. This seems to indicate that the sequence of his narrative at this stage likely tracks his actual course of investigation, for he had reached an impasse in seeking to complete his list of necessary and unnecessary conditions. To identify conditions that could be associated with the different colours, he had probably looked first to the angles which the edges *EH* and *DF* of the illuminated region make with the prism bottom, but this did not work because the order of angles can be different while the colours remain the same. Stymied by the apparent absence of difference where it ought to be, Descartes recurred to mechanism for a solution.

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[38] PO, 336; AT, v6, 331.
[39] PO, 332; AT, v6, 325.

'Conceiving the nature of light to be as I described it in the *Dioptrique*, namely as the action or movement of a certain very fine material ... ', Descartes anticipated, would open a route to the deduction of the conditions of difference.[40] In the *Dioptrique*, Descartes had already, and later (in)famously, outlined ways of thinking about refraction in terms of mechanism. I leave aside for present purposes the difficulties that puzzled many of his readers who tried to understand how light could be treated as a 'tendency' to motion and yet written about in ways that seemed to invoke bodily motion proper. I turn instead to Descartes's 'fine material', his little spheres, whose pressing upon one another constitutes the mechanical essence of light itself. We have two media in both of which small balls are interspersed among larger particles; the latter differ in size between the two media. Depending on the nature of the two media, one of several things may occur at their bounding surface when the little balls in one of them are pressed towards the other.

Descartes had discussed the possibilities in the 1st Discourse of the *Dioptrique*.[41] There, he had invoked the analogical behaviour of macroscopic bodies. If the surface is 'soft', then balls striking it are stopped, their motion stifled as though 'thrown into linen sheets, or sand, or mud'. If hard, the balls deflect, but in one of several ways. If the surface is very flat, they are deflected but keep marching in order. If it is curved, they splay out at angles but keep order among one another according to the curvature, but if the surface is rough, something new occurs to the balls: if they 'had beforehand only a simple straight movement, they lose part of it, and acquire instead a circular motion'—on the analogy of a tennis ball hit with a 'cutting or grazing' action. When this occurs, the now-spinning balls produce colours. A coloured body puts the correlated spin on the balls which reflect from it.

Descartes's scheme for colours was soon discussed in letters to him by the Dutch Jesuit Jean Ciermans (1602–1648) and the French natural philosopher and astrologer, the monumentally verbose Jean-Baptiste Morin (1583–1656), who pointed out what they thought to be mechanical inconsistencies.[42] As they and others discovered, it is difficult to grasp just how the tendency to motion of Descartes's little spheres works together with the colour-producing mechanism that he envisioned. There is ample room for confusion here, as in many other areas of Descartes's scheme, but at least one thing is certain: he aimed to link differences in spin among the small balls to observational conditions, thereby producing a criterion for establishing order among the colours. To do so, Descartes deployed two factors: the boundary between the lit and the dark, and the refracting interface proper. To understand his reasoning, which mingles conditions for colour generation with invisible mechanisms, it is particularly essential to emphasize that Descartes was working with finite-width beams and not with line-like paths.

In Descartes's Figure 5, the region within the dotted lines represents a beam of light passing through air which strikes the surface *YY* of water. The air consists of larger particles between which the fine matter—the small spheres—is insinuated. When pressed, these spheres tend to motion—Descartes obfuscated the difference between tendency and motion at this point by referring to the particles' 'action or movement'. If the beam contacts the surface head on, then all of the spheres

[40] PO, 336; AT, v6, 331.
[41] *Dioptrique*, First Discourse: AT, v6, 89.
[42] Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes*, 212–18 discusses Ciermans and Morin. We will return below to these critiques and Descartes's response to them.

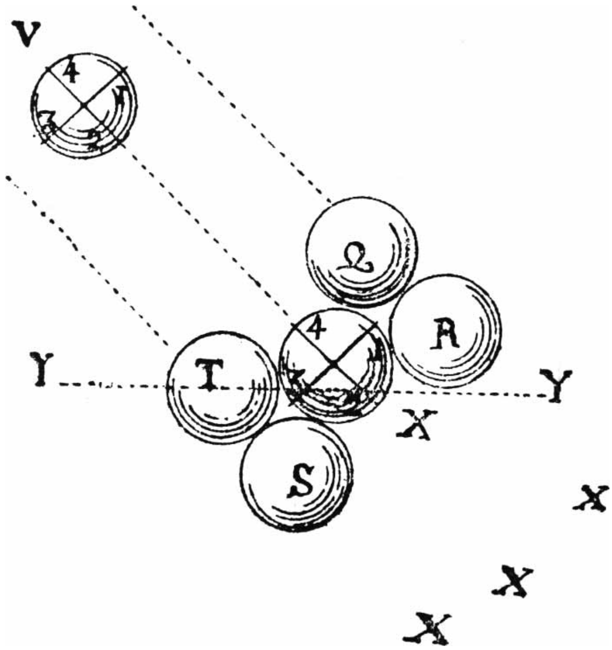

Figure 5. Descartes's refracting spheres.

communicate their tendency in precisely the same way to their siblings within the region that contains the hard (compared with air) particles of water. If, on the other hand, the beam contacts the surface at an angle, then a difference emerges. Descartes accordingly sought to examine the effect produced by an oblique beam by elaborating the assertion in the *Dioptrique* that the small balls are capable of rotation, extending it here to the circumstances of refraction: they 'must be imagined just like small balls that roll in the pores of earthly bodies'.[43] Furthermore, they can 'roll in various ways according to the various causes which determine them'.[44]

The production of colour depends upon generating differences, founded on the assertion that the spheres have a normal rotational movement or tendency whose magnitude (regardless of its direction) engenders white. Any deviation from the usual magnitude produces colour, which is accordingly a differential situation. The fulcrum of the account consists of a link between rotational speed (or tendency) and motion (or tendency) in a straight line. The spheres at the coloured edges of the prism beam (Figure 3) must have either a 'stronger [or else a weaker] tendency to rotate than to move in a straight line'. If the tendency to rotation is the same as that to move straight—the 'usual' situation—then no coloration occurs. This raises the question of

[43] PO, 336 (translation altered); AT, v6, 331. The original reads 'il faut imaginer les parties ainsi que de petites boules qui roullent dans les pores des cors terrestres'.

[44] In the ninth Discourse of the *Météores*, Descartes remarked that 'the normal movement of the small particles of this material—of those in the air around us, at least—is to roll in the same way that a ball rolls on the ground, when it is propelled only in a straight line. And it is the bodies that make them roll in this way which we properly call white' (PO, pp. 346–47; AT, v6.). Westfall, 'The Development of Newton's Theory of Color', 341 notes the point concerning white.

how Descartes envisioned the creation of a difference between the 'usual' situation and situations which involve colour—or indeed what it means to say that a rotational motion might be stronger or weaker than one in a straight line.

To suggest what Descartes might have had in mind with this curious comparison between linear and rotational motions, I will follow his own procedure. Like him, I will deploy a macroscopic comparison, as when Descartes had likened the engendering of a rotation in the little spheres by a coloured surface to the cutting action of a racket on a tennis ball. Let us compare a billiard ball rolling on the rough felt surface of a table to Descartes's 'small balls rolling in the pores of earthly bodies'. If the billiard ball has not been hit too hard, then it will roll on the table without slipping, which establishes a direct link between its rotational speed about the point of contact with the table and its translational speed. If, per contra, the ball is hit hard, the connection between rolling and translating will be broken, and it will slip along the surface while spinning. Anyone who has spent a moment watching or, even better, playing billiards has seen or experienced the effect. White light then corresponds to the state of the beam when the (tendency to) rotation of the spheres is coupled to their (tendency to) translation; colours arise when the two are decoupled. The 'usual' ratio of rotation to translation that obtains with non-slip rolling then constitutes the ratio from which coloured light must depart, with colour depending only on the magnitude, not the direction, of the rotation. This comparison to macroscopic rolling, as with all of Descartes's micro-macro similarities, is analogical, depending only on the occurrence in both situations of motions of the same kind in figures of the same shape—a point that understandably confused Morin, and no doubt many other readers as well.

Figure 6 illustrates an uncoloured, columnar Cartesian beam of incident light whose little spheres all (tend to) rotate with a magnitude equal to what they would have if they were to roll without slipping, thereby unambiguously coupling the rotational to the linear movement. The direction of their rotation in the white beam is irrelevant; only the magnitude determines the 'usual' state that will engender white. For simplicity, I have drawn the spheres rotating clockwise, which would be the case if they actually did translate towards the interface as a result of roll without slip. At the beam's left edge, a sphere at the interface is subject to a differential action from its neighbours that increases the magnitude of its rotational speed above the 'usual', producing red. As we move towards the centre of the beam, the differential decreases because the neighbouring spheres are increasingly in the same situation, producing

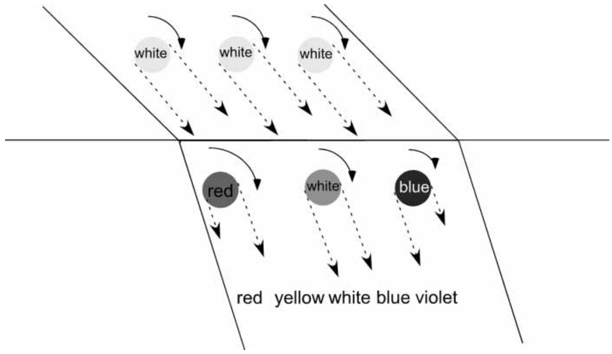

Figure 6. Cartesian generation of prismatic colours.

yellow, until at beam centre the neighbouring actions are precisely the same, thereby leaving the sphere's usual rotation unaltered and so producing white. Moving further on, towards the beam's rightmost edge, the differential reverses, producing first a green and then blue, and finally, in what Descartes called 'normal' circumstances, his 'rosy' blue, or violet.

Although Descartes's exposition does a reasonable job explaining the origins of an increased rotation at the beam's left edge, he used the same diagram (Figure 5) for its decrease at the other edge, which makes the mechanical analysis somewhat obscure. My Figure 7 breaks apart the two edges in order to clarify the situation. The leftmost set of spheres correspond to Descartes's Figure 5 and represent the rotation-increasing configuration. With Descartes, I divide the affected sphere into labelled quadrants and surround it with four neighbours. Two of these (*Q* and *R*) are toward the beam centre and fully within the incident light, while the other two (*T* and *S*) are toward the left edge and at least partially within the refracting medium. These latter two are said not to move with as much 'force' as the other pair. Sphere *S* acts in effect as a stationary pivot for part 3 of the affected sphere, tending to hold it in place. *Q*, however, twists part 1 clockwise, increasing rotation. *R* moves away and so has no effect, while *T* lags and also has no effect. At the right-hand edge, the situation is, according to Descartes, mechanically reversed in respect to the actions of the four spheres on the one between them: *q* and *s* now do nothing, while both *r* and *t* decrease rotation as *r* impedes clockwise twist, while *t*, moving 'faster' or with more 'force'—he used the two words interchangeably here—twists counterclockwise.

The entire scheme rests on a presumptive difference between beam edges, but Descartes had not so far provided a way observationally to distinguish which edge of the beam increases rotation, and which edge decreases it—and so which edge produces which colour. He could not rely on the respective angles at which the edges emerge from the prism, because he had observed that the colours remain in the same loci on the screen even when the relative order of the angles reverses. Another criterion had to be found. If it were the case that the spheres in the incident white beam did always rotate in the same direction that a rolling ball would, then the beam sides could be distinguished by determining on which edge a sphere would have to roll in order to move forwards, for then the rotation at that edge would be increased

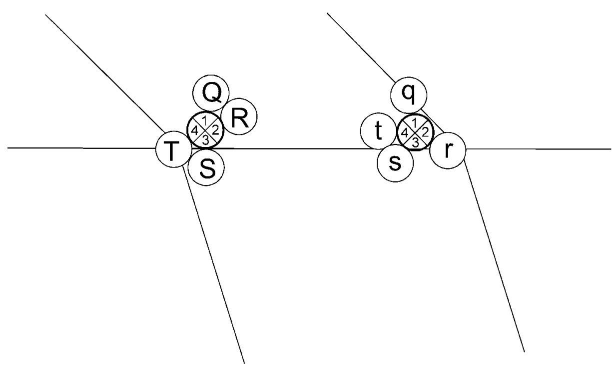

Figure 7. Cartesian mechanics at beam boundaries.

by the neighbouring light within the beam, and it would be decreased at the other edge since the spheres there would be twisted in the opposite direction. However, Descartes rejected the idea of uniform rotational directionality in his explanations to Ciermans, as well he should have, since random effects in the passage of light even through air militated against it.[45]

5. Pressed Spheres and a 'Rosy' Blue Suggest How to Generalize the Notion of an 'Aperture'

Descartes had to this point worked within the confines of his imagined world of little spheres without mentioning the dispositions of colour. One might think that his next step would be to associate red with the effects at one edge, and blue with effects at the other—though Descartes has thus far not observationally distinguished the edges from one another. But he did not forge the association, or rather not immediately, for he first detoured through an exception to the mechanics that he had been describing. Suppose, Descartes imagined, that balls *r* and *t* press 'fairly hard' on the one between them. Then, they will not just decrease the rotation; they will actually cause the sphere to rotate in the opposite direction at a good clip—presumably because *r* now acts as a fixed pivot, allowing *t* free reign. If the induced rotation is speedy enough, the result will be to produce a situation at the right edge that is mechanically similar to that at the left edge. Although this probably was an ad hoc adjustment to account for the observational discrepancy which immediately follows in his text, it can also be read, I will now argue, as a productive stimulus that led to the solution of an urgent difficulty.

Descartes referred to the presumptive rotation-reversal that occurs when *r* and *t* press sufficiently hard as having enabled him 'to resolve the most important of all the difficulties that I had in this matter'. He did not specify this 'most important' difficulty but followed the statement with the explicit introduction of coloration, whereby red is associated with increased, and blue with decreased, rotation.[46] Immediately thereafter, he invoked the apparent violation of this order by his 'rosy' tint: the blue coloration visible in the beam of Figure 4 (left), Descartes interjected, is 'normally ... mixed with a rosy colour at its edges, which gives it vivacity and glitter and changes it into a violet or purple color', which implies rotation augmented above, instead of decreased below, the usual.[47] To deflate the anomaly, Descartes linked the infringement of colour order to the mechanical exception that he had just described.

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[45] Descartes explained to Ciermans that 'all these little globes contained within the pores of glass, air, and other bodies always, or at least most often, have an inclination or propensity to turn in some direction, and even to turn with a speed equal to that with which they are moved in a straight line, as long as they do not encounter a specific cause which augments or diminishes this speed ... most of them have different inclinations, according to their diverse encounters with the confines of the pores where they are located; so that if some among them incline to turn to one side, others incline to turn at the same time to another' (Clerselier, ed., *Lettres De Mr Descartes*, 3 vols. (Paris, 1724), 298–99). He follows this with an example that insists on the random directional effects of encounters with large particles. Descartes seems not to have thought that these encounters would alter the rotational tendencies per se but only their directions, the motion proper being conserved except in the special cases of coloured surfaces and at the edges of shadowed regions in refraction. For Descartes, a rotating sphere would necessarily continue to rotate unless forced not to for the same reasons that undergird his famous analysis of a stone whirled 'round in a sling: the glue that holds the parts of the sphere together pulls them back from the innate force that, as he understood the dynamics, tends to move them outwards in order to remain along the path's tangent.

[46] This establishes a chromatic scale running from red through white to blue, but the Cartesian scale applies only to prismatic colours and furthermore has no clear implications even for the mixing of lights.

[47] As noted in Westfall, 'The Development of Newton's Theory of Color', 343.

The narrative accordingly places the explanation *before* the observation, and this gives the impression that the exception had occurred to him, as it were, from within the logical body of the mechanism, which he then naturally drew on to handle an otherwise anomalous situation. Although we can be quite certain that the explanation followed on the observation, it may nevertheless have been Descartes's mechanistic ruminations about the 'rosy' tint that suggested a route to the rainbow's colours. His 'most important of all the difficulties' may accordingly have a twofold signification. Given its position in the narration, the statement refers on the one hand to the rosy anomaly; on the other hand, it also refers to the gravest problem of all, namely to discover in the raindrop something like the colour-generating aperture of the prism.

Descartes's seemingly offhand remark that the 'rosy' tint only appears 'normally' enfolds a critically important insight. To see why, let us return to my reproduction of his experiment with the prism. The sole factor that can be adjusted is the width of the aperture, and I observed that the rosy tint appears only when it becomes very small—in effect, a pencil-point-sized hole. As the aperture grows, the tint is replaced by a solid blue line (Figure 4 right). The effect is quite marked and cannot be missed, even in casual observation. And so by 'normally', Descartes likely meant with a sufficiently small aperture. Here, I suggest, he spied a clue that brought the effect of aperture width on coloration to the forefront of his considerations.

At the outset of his experimental work with the prism, Descartes had merely noted the effect of width on the extent of coloration. As the width grows, the colours retreat to the edges, becoming as it were reduced to edge-hugging red and blue. As he initially worked through his mechanism of small spheres, Descartes focused on the interactions among them at an aperture-delimited refracting surface. However, at this early stage, the interactions were effectively independent in kind of the aperture width. That is, a greater width would weaken the sphere-to-neighbouring-sphere interaction, but the nature of the effect did not change—nothing in the mechanism suggested that it should. The aperture accordingly recedes from view in the deductive scheme, since it has a fixed outcome.

The rosy tint brings the aperture to the centre of interest. It occurs only when the opening is sufficiently narrow: when the appropriate size is reached, what had been a solid blue becomes reddish. In Descartes's mechanical scheme, this meant that the effects of the neighbouring little spheres on those near the affected edge have markedly altered, and the only factor that could be involved was the narrowed gap. If the rosy tint had not occurred, per contra, the aperture would have remained in the background, since its action would have been a constant expansion of coloration with decreasing width without any change in the order of colours once they become apparent: only differences, not invariant effects, permit the selection of alternate paths in Descartes's idealized deductive scheme.

The appearance of the rosy tint made just the sort of alteration to which Descartes would have been alert, and this meant that apertures must play important roles in the selection of an apposite deductive path. It could not, however, be the body of the aperture per se that was at issue, because it never changed and as such could not make a difference. What did change was the size of the space between the aperture's borders, a region in which its physical edges are absent. This shifts attention from the aperture's body to its effects on whatever takes place within the illuminated region. One might thereby begin to think of an 'aperture' not as something which always blocks light but as *any cause that alters the interactions among the light-bearing entities in the illuminated region—Descartes's small spheres—from one place to the next*. With physical apertures, the change is abrupt at the edges, where light cuts out, but it is more gradual between them, where the magnitude of the aperture's width determines the rate at which the change occurs. Sometimes it occurs slowly from point to point (within wide apertures), sometimes more rapidly (within narrower ones), and sometimes swiftly, with concomitant changes in the results of the interaction (within very narrow ones, with their vivid, distributed coloration and otherwise anomalous rosy blue at an edge). Coloration accordingly arises wherever the interactions change from place to place, though if the change is very gradual, colours will not be visible, and if extremely rapid, an apparent modification of colour order occurs. It is consequently possible, and even probable, that the anomalous rosy blue set Descartes on the path to reconceptualizing the notion of 'aperture'. And since the spheres demarcate the paths—the rays—of light, it follows at once that coloration is intrinsically linked to changes in the interactions among neighbouring *rays* from one region to the next. Note the important connection: rays of light may affect one another precisely because they are associated with Descartes's small spheres, so that the possibility of interactions among rays in the production of colour arises directly out of simple mechanical reasoning.

According to this chain of thought, the rosy tint by itself was not what led Descartes to a functional generalization of the aperture. Not at all—the role of the anomalous tint was to recentre Descartes's attention on the reciprocal effects of the small spheres. In the absence of his mechanical model, it seems highly unlikely that Descartes would have known what to do with the rosy exception—recall his assertion that, in reconsidering the interactions ('it must be noted . . . '), he had decided that an augmentation of rotation at a boundary where diminution usually occurs 'can easily happen' if the pressing of a sphere on its neighbour is exacerbated. It was that realization which enabled him 'to resolve the most important of all the difficulties that I had in this matter'. Here, it seems, we have what may be a unique instance in which a Cartesian mechanism actually played a *generative role* in producing a result with observational significance (the transformation of the 'aperture' from a physical blocker of light to a cause that alters interactions in an illuminated region), a result that had potential implications beyond the specific phenomenon that the mechanism had been designed to accommodate. Bluntly put, Descartes could now *predict* that coloration should appear wherever the concentration of light rays becomes extreme. The next step would be to see whether this occurs in the case of the raindrop. Further, if I am correct then Descartes was engaged in a *process of exploratory testing* to see whether an implication drawn from one experimental regime (the prism), and tied closely to his mechanism, would apply in another (the rainbow) whose properties had not yet been investigated but which had to involve similar mechanical interactions.

6. The Clustering of Rays at a Cutoff Angle Simulates an Aperture Edge

'In all of this', Descartes remarked with evident satisfaction at this stage of the narrative, 'the explanation accords so perfectly with experience that I do not believe it possible, after one has studied both carefully, to doubt that the matter is as I have just explained it'. And yet the two central issues are still unresolved: what accounts for the *production of colours* by a raindrop where there is no evident aperture, whether wide or narrow, and furthermore what explains the specific *order of colours* in the primary and secondary bows? We will turn presently to the question of order, but first we will follow Descartes's explanation of why colours appear at all when sunlight passes through a raindrop where there seems to be no aperture to produce the differential effects that alter the rotations of his small spheres from the 'usual' state. 'I doubted at first', Descartes wrote, 'whether the colors were produced [in the rainbow] quite in the same way as in the crystal *MNP*; for I did not notice any shadow which cut off the light'. Descartes's exploration of prismatic colours had provided him with an initially puzzling, but eventually exemplary, situation that, I argued, suggested a means to evolve the function served by the aperture in a way that would naturally attribute similar effects to a change from place to place in the interactions among his small spheres. A physical aperture produces such a change without altering the amount of light through a given region—a quantity that, we will see, Descartes would measure by tabulating the distribution of a given number of initially parallel rays that are refracted or reflected into the region. However, if this number, the ray density as it were, alters from one place to the next for any reason then that too implicates a spatial change in the interactions among Descartes's little spheres.[48] This accordingly ties the question of colour in the absence of a physical aperture to places where rays cluster, which must then be the cause of the rainbow's coloration. To find out whether, and if so where, this takes place, ray paths had to be traced through the bow-producing raindrop.

In Figure 8 right, which is based on Descartes's original (Figure 8 left), a ray *AF* enters the water-filled flask—his artificial raindrop—and after reflection at *K* either emerges at *N* or is again reflected, to emerge at *Q*. Descartes measured the angles ∠ONP and ∠SQR, respectively, for one and two reflections. These are Descartes's viewing angles. He always worked with circle arcs and line lengths and did not compute with the angles of incidence and refraction *i,r*, which are ∠CFK, ∠CFG, but the relations are obvious enough since the arcs $\widehat{FK}$, $\widehat{FG}$ are, respectively, 180°–2∠CFK, 180°–2 ∠CFG.[49] The circle's symmetry made computations particularly simple—much simpler than Descartes's earlier geometry for refraction through a prism in the *Dioptrique*—because equiangular internal reflections mean that the same angle of refraction obtains at every reflection, and so a ray always emerges at the angle at which it was originally incident. Further, since *FH* measures the sine of incidence, and *CT* the sine of refraction, then *FH*/*CT* has the same ratio for all rays. At each reflection, the ray deviates through equal arcs $\widehat{FK}$, while at entry and emergence, the deviations are always *i*–*r*, i.e. $(\widehat{FK}-\widehat{FG})/2$. The viewing angle is measured by the total deviation of the emergent ray from its direction of entry into the droplet. For singly reflected rays, the total deviation will be $2\widehat{FK}-\widehat{FG}$, and so the viewing angle must be $180^\circ+\widehat{FG}-2\widehat{FK}$. Similarly, the total deviation for double reflection will be $3\widehat{FK}-\widehat{FG}$, which yields a viewing angle of $3\widehat{FK}-\widehat{FG}-180^\circ$.

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48 There are, however, potential difficulties in forming a consistent understanding of optical intensity in Descartes's scheme if ray counts are to be used: see below, note 58.

49 Descartes accordingly had to perform the following computations. Given the refractive index, *n*, the drop radius *R*, and the distance *FH,* the corresponding arcs $\widehat{FK}, \widehat{FG}$ are, respectively, $2\sin^{-1}(\sqrt{1-(FH/nR)^2})$, $2\sin^{-1}(\sqrt{1-(FH/R)^2})$. If—like Harriot before him—Descartes had worked with angles of incidence and refraction, then he would instead have computed *r* as $\sin^{-1}(\sin(i)/n)$ followed by simple additions and subtractions to obtain $\widehat{FK}, \widehat{FG}$. Descartes's method requires two trigonometric look-ups together with squarings and square roots.

A treatise probably written by Benedict de Spinoza (1632–1677), but printed posthumously without an author's name in 1687, provided the tiresome details that Descartes left to the reader. 'As is his wont', Spinoza wrote, 'he [Descartes] simply presents his table, without revealing to those interested in algebra how he discovered the two laws of refraction by means of which he worked it out' (M.J. Petry, ed., *Spinoza's Algebraic Calculation of the Rainbow & Calculation of Chances*, vol. 108 (Dordrecht, 1985), 39). Spinoza proceeded to introduce *x*, *y* to represent ∠ ONP, ∠ SQR in order to 'reproduce it in algebraic terms' (p. 53).

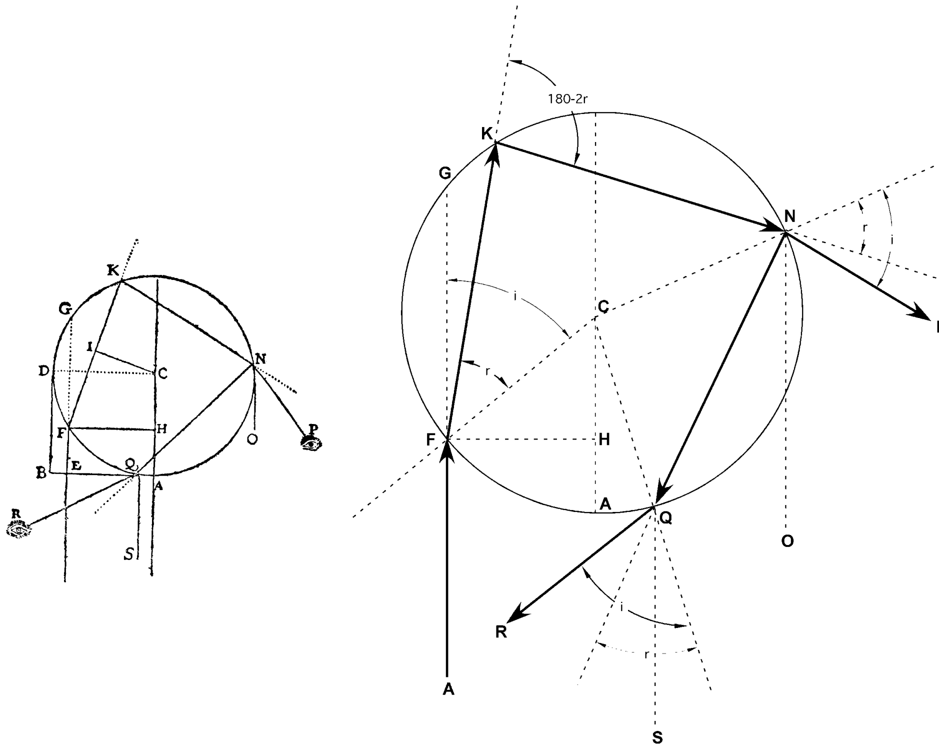

Figure 8. Singly and doubly reflected rays within a spherical droplet: Descartes's figure (left) and an adaptation of it (right). Ray *AF* enters the drop and is refracted at *F* to *K*, where it is reflected to *N*. At *N* the ray both emerges as *NP* (producing the primary bow) and is reflected to *Q*, where it emerges as *QR* (producing the secondary bow).

The sun's light bathes a raindrop in a group of parallel rays, and Descartes had therefore to choose an appropriate set among them for his geometry. The selection could in principle be done arbitrarily, but Descartes chose to space his rays at equal linear distances apart, beginning at the centre of the drop and proceeding to its edge. He might instead have chosen another method of selecting among them, one that is equally arithmetic: he might, that is, have chosen his incident rays to strike at arithmetically increasing angles of incidence, which will not space them equally at all. Harriot had done just that, and we will see that working in these two different ways opens different possibilities. But let us first examine what does occur with Descartes's equably spaced rays.

Figure 9 depicts both types of emergent rays—those that are singly, and those that are doubly, reflected within the droplet for Descartes's index of refraction for water (250/187)—a curiously precise value which he claimed to have measured.[50]

[50] At 20°C, sodium light (589.3 nm) has an index of 1.33299, which reaches 1.33348 at 14°C. Descartes's value, 1.3369 to four decimal places, is too high for the central spectrum, which perhaps indicates (assuming he did measure) that he observed a dispersed beam and took the index towards its most refracted terminus. Presumably, Descartes would have used the apparatus he described in the *Dioptrique* (PO, 162; AT, v6, 212), though without reconstructing the device it is difficult to tell how accurately an index can be measured with it.

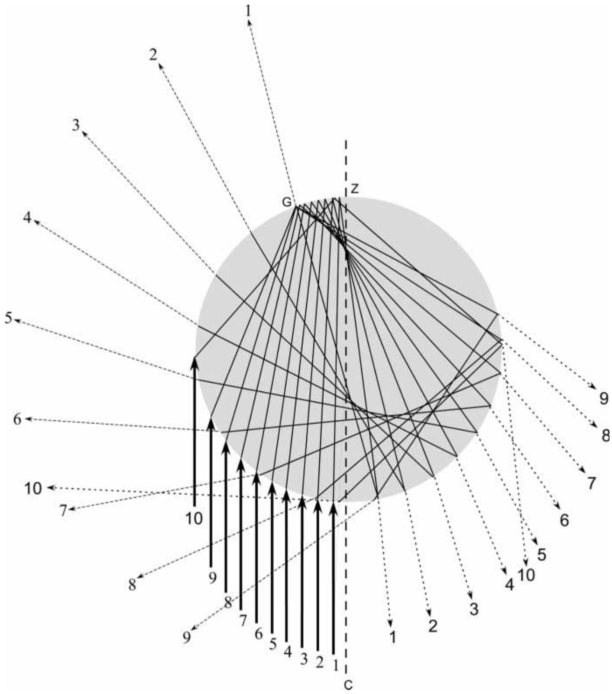

Figure 9. Ray paths within a droplet. Descartes's diagram (figure 8 left) depicts the paths of ray 9 here.

Here the incident ray, remaining always parallel to a given direction specified by a line through the droplet's centre, moves outwards in equal steps from striking directly along the central line to just grazing the circumference.[51] Any given ray emerges at two particular points along the circumference (viz. N or Q in the figure). Singly reflected light occurs only in the region between about 250° and the bottom, while doubly reflected light is absent between about 335° and (moving clockwise) the top.

[51] All of the first-refracted rays are internally reflected from a comparatively small arc at the top of the sphere from G to the point Z that lies on a diameter along line CZ which is parallel to the rays of the incoming set. Isaac Barrow (1630–1677) determined that (in our figure 9) the counterclockwise limit (G) of the region marks the point at which the intersection of a pair of indefinitely close refractions lies on the surface of the sphere itself. From this, it followed without the computation of an extremum that the viewing angle must be a maximum: see Alan E. Shapiro, 'The Optical Lectures and the Foundations of the Theory of Optical Imagery', *Before Newton. The Life and Times of Isaac Barrow*, edited by M. Feingold (Cambridge, 1990), 144–47 for Barrow on the rainbow. At this angle, neighbouring rays among an initially parallel set accordingly remain parallel to one another on emergence; at other viewing angles, they diverge. In figure 9, rays 8 and 9 remain nearly parallel, whereas all other ray pairs are divergent.

The loci of emergence for both singly and doubly reflected rays move counterclockwise with increasing incidence until certain angles are reached, at which point the loci reverse direction. The two angles differ, since the respective positions of N and Q are $2\pi - 4r + i$ and $3\pi - 6r + i$. Consequently, the angles for reversal are, again respectively, $\sqrt{(16-n^2)/15}$ and $\sqrt{(36-n^2)/35}$ or, for Descartes's index of 250/187, about 76° 45′, 81° 23′; the corresponding radial distances FH are (in Descartes's units) 9734, 9887. Although the last entry in each of Descartes's two tables could be used to show the reversal, he did not list the actual position of the points of emergence N and Q because he did not tabulate the angles $\widehat{FA}$ of incidence (figure 11). Below the reversal angle, the singly reflected emergent rays project back to intersect at the point Z in figure 9. No such point exists for doubly reflected rays.

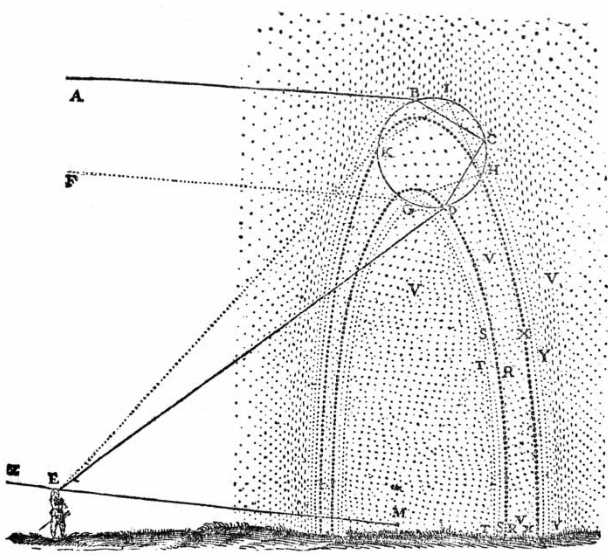

Figure 10. Descartes's depiction of the primary and secondary bows.

Consequently, the two types of ray occur together only in the region from about 335° to the bottom. Descartes's own diagram (Figure 8 left) displays a singly reflected ray emerging at *N*, and a doubly reflected one emerging at *Q*.[52]

Descartes had observed that the light at the flask's bottom is brightest and coloured red at a viewing angle of about 42°, above which it cuts off, and the light from the flask's top is brightest and red at about 52°, but less so than at the bottom, below which it too cuts off (Figure 10).[53] To see whether the effects correspond to ray clustering, as the expanded concept of the aperture would suggest, Descartes computed two tables, in effect tracing the paths of select rays (Figure 11).[54] Setting the radius of the droplet to 10,000 (and so computing the linear distances to five significant figures), and taking the incident rays to form an equably spaced set starting (for the first table) at 1000 units from the central line QC, and increasing in steps of 1000 to grazing incidence at *DB* (Figure 8 left), Descartes then computed the distances *CI*, the arcs $\widehat{FG}$, $\widehat{FK}$, and finally the viewing angles $\angle ONP$, $\angle SQR$.

Since the viewing angles vary continuously with the distances of the incident rays from the central axis, we would today conclude (as did Newton a quarter-century later) that a small change in the viewing angle near its extremum contains more rays than anywhere else, from which it is immediately obvious that the droplet will appear brightest at these extrema. Here, one might think, Descartes might usefully have

[52] Their loci can be computed using Descartes's tabulated arcs (see below, figure 11) from a corresponding table of the $\widehat{FA}$ (though Descartes did not provide one), since these latter are the angles of incidence, $\sin^{-1}(FH/R)$. Then, moving clockwise, *Q* is located at $\widehat{FA} + 3\,\widehat{FK}$, while *N* is located at $\widehat{FA} + 2\,\widehat{FK}$.

[53] '... its part *D* appeared to me completely red and incomparably more brilliant than the rest ... this part *K* would appear red too, but not as brilliant as at *D*'.

[54] There are three effects here: extreme brightness, coloration, and cutoff. Only the first in and of itself suggests ray clustering.

LA LIGNE HF	LA LIGNE CI	L'ARC FG	L'ARC FK	L'ANGLE ONP	L'ANGLE SQR
1000	748	168.30	171.25	5.40	165.45
2000	1496	156.55	162.48	11.19	151.29
3000	2244	145. 4	154. 4	17.56	136. 8
4000	2992	132.50	145.10	22.30	122. 4
5000	3740	120.	136. 4	27.52	108.12
6000	4488	106.16	126.40	32.56	93.44
7000	5236	91. 8	116.51	37.26	79.25
8000	5984	73.44	106.30	40.44	65.46
9000	6732	51.41	95.22	40.57	54.25
10000	7480	0.	83.10	13.40	69.30

LA LIGNE HF	LA LIGNE CI	L'ARC FG	L'ARC FK	L'ANGLE ONP	L'ANGLE SQR
8000	5984	73.44	106.30	40.44	65.46
8100	6058	71.48	105.25	40.58	64.37
8200	6133	69.50	104.20	41.10	63.10
8300	6208	67.48	103.14	41.20	62.54
8400	6283	65.44	102. 9	41.26	61.43
8500	6358	63.34	101. 2	41.30	60.32
8600	6432	61.22	99.56	41.30	58.26
8700	6507	59. 4	98.48	41.28	57.20
8800	6582	56.42	97.40	41.22	56.18
8900	6657	54.16	96.32	41.12	55.20
9000	6732	51.41	95.22	40.57	54.25
9100	6806	49. 0	94.12	40.36	53.36
9200	6881	46. 8	93. 2	40. 4	52.58
9300	6956	43. 8	91.51	39.26	52.25
9400	7031	39.54	90.38	38.38	52. 0
9500	7106	36.24	89.26	37.32	51.54
9600	7180	32.30	88.12	36. 6	52. 6
9700	7255	28. 8	86.58	34.12	52.46
9800	7330	22.57	85.43	31.31	54.12

Figure 11. Descartes's tables for the refractions of equably spaced, parallel rays by a water sphere.

deployed his procedure for representing curves by means of the distances of a point from two lines at an angle to one another. A purely graphical representation of this sort does not require generating a corresponding equation for the resulting curve. Nevertheless, Descartes always considered the curve to be a primary entity of which only certain types were suitable for representation in this way. The curve had, namely, to be producible by means of a 'continuous' tracing motion of a certain type; it may be compound—due that is to several motions—but each one in the sequence must be completely determined by its predecessor. More to the point here, these several motions must have mutually commensurable speeds. This, Descartes knew but did not demonstrate, confined the structure to the class of algebraic curves. However, the curve generated by setting one right line for the *FH* distances and the other for the viewing angles would not satisfy such a condition because it involves trigonometric properties. Moreover, the use of curves to encapsulate a relationship between two variables, whether mathematical or otherwise, remained uncommon until the beginning of the nineteenth century, though Edmund Halley (1656–1742) graphed barometric pressure against altitude in 1686, and Christiaan Huygens (1629–1695) a distribution function in 1669.[55]

[55] Edmund Halley, 'A Discourse of the Rule of the Decrease of the Height of the Mercury in the Barometer, According as Places Are Elevated above the Surface of the Earth, with an Attempt to Discover the True Reason of the Rising and Falling of the Mercury, Upon Change of Weather', *Philosophical Transactions* 16 (1686) 104–15. For Huygens, see Anders Hald, *History of Probability and Statistics and Their Applications before 1750* (Hoboken, NJ, 2003), 108–109; the graph appears in Christiaan Huygens, *Oeuvres Complètes De Christiaan Huygens*, edited by Société_Hollandaise_des_Sciences (La Haye, 1888–1950), 526–31, vol. 6.

Table 1. Values at the outer primary and inner secondary bows.

	Singly reflected	Doubly reflected
Angle of incidence (*i*)	59° 11′	71° 43′
Angle of refraction (*r*)	39° 58′	45° 15′
Radial distance (*FH*)	8573	9480
Extreme viewing angle[a]	41° 31′	51° 58′

Note:
[a]These are uncorrected for the Sun's radius, on which see note 56.

Descartes reasoned directly from his tabulated angles. His first table indicated that $\angle ONP$ seemed to reach a maximum of about 41°, and $\angle SQR$ a minimum of about 54°, in the vicinity of incident rays that, respectively, strike at 8000 and 9000 units from the central line. This encouraged him to press further by generating a table at intervals of 100 units from 8000 through 9800. The 'more precise' table for pinpointing the extrema now set them respectively, at 41° 30′, 51° 54′ at incident displacements of 8500 or 8600 and 9500. To the minimum, Descartes added, and from the maximum, he subtracted, 'around 17 minutes for the radius of the Sun',[56] producing final results of 41° 47′, 51° 37′.

Descartes's table accordingly generated values for the cutoff angles that squared reasonably well with his water-flask observations of 'around' 42°, 52°. The precision of the tabulation led him to indulge a smugly satisfied slight at the expense of Maurolycus, whose values of 45°, 56° 'shows how little faith we must have in observations which are not accompanied by true reason'—reason here referring to geometric computation, which for Descartes trumped unassisted observation. Not for him the Baconian ingathering of matters of fact. Table 1 provides accurately computed values for the extremal viewing angles,[57] as well as for the corresponding incidences, refractions, and distances *FH* for Descartes's presumptive index of 250/187 and his drop radius of 10,000 units.

To explain the vibrant coloration of the rainbow, Descartes needed more than a simple cutoff, for that also occurs in the border-limited coloration of wide apertures produced by the prism; he needed to find a simulacrum of the narrow aperture. His tables did not, strictly speaking, provide one, because both primary and secondary bows were lit through a considerable range past their respective cutoff points. It is precisely here that Descartes was able to evolve the notion of the aperture's function by using the anomalous observation of the 'rosy' violet as a fruitful resource: it had implied, in his understanding, an increased pressure on the spheres, which he apparently now considered to be the primary operative factor in the generation of a full-blown coloured region. A simple cutoff would produce colours near a border, as with a wide aperture, but only an increase in the interactions between neighbouring rays (balls) could fill a region with tints. More light in a given region would have the same effect. In Descartes's words, 'not receiving rays of light in your eyes*, or receiving*

56 Although the angle which the extreme ray forms with a line perpendicular to the horizon depends only on the index of refraction, the Sun's angular extension does affect the viewing angle $\angle pEM$ (Figure 10). The tabular computation assumes rays from the Sun's centre and must accordingly be adjusted.

57 The singly and doubly reflected extremal angles $\angle ONP$, $\angle SQR$ are, respectively, $\sin^{-1}(\sqrt{(4-n^2)/3})$, $\sin^{-1}(\sqrt{(9-n^2)/8})$.

notably fewer of them from one object than from another that is near it [emphasis added], is the same thing as seeing shadow'. And indeed he had his result, because 'there are many more rays' near the cutoff angles than elsewhere.[58]

From a later standpoint, the very existence of a minimum or a maximum angle entails a concentration of rays because at an extremum, a comparatively large change in the incidence produces a small change in the viewing angle, thereby collecting rays in its vicinity. Descartes did not directly make the connection, for he noted first that the rays concentrate near two angles and then, separately, that this occurs near cutoffs.[59] Neither did he deduce that the extremum, in the case of the primary bow, occurs where the tangent of the angle of incidence is twice the tangent of the angle of refraction. And yet Harriot had much earlier reached precisely that result, having himself obtained a form of the law of refraction and applied it to the rainbow.

Johannes Lohne discovered the relationship, uncommented and (like the law of refraction itself) unpublished, in Thomas Harriot's (1560–1621) manuscripts. Of it, Lohne remarked in 1965 that 'this tangent proportion (for the primary bow) can scarcely be found without (direct or indirect) infinitesimal considerations'.[60] How, then, did Harriot, who did not have infinitesimal methods available, find it, and why did Descartes not do so?

Harriot left no explanation, but, based on the manuscripts, Lohne provided a persuasive route that he might have followed, one which moreover shows just why Descartes would not have uncovered the relation.[61] Harriot, like Descartes, computed a table. His was, however, different in that Harriot tabulated for

[58] Another possible difficulty with Descartes's scheme, though not one that seems to have occasioned much if any comment, concerns optical intensity proper. Descartes here associated it with the number of rays in a given region, implicitly assuming that each ray has the same, or effectively the same, visual effect. Optical intensity in the Cartesian mechanical model can presumably be changed in only two ways: either the originating luminous pressure is altered, or the number of spheres in a given region that are subject to the pressure is itself changed. Descartes's world is always completely full, and so to alter the ball numbers requires that they displace the larger particles of air, for example. However, the number of rays can be increased without doing anything directly physical to a region and without changing the originating pressure at the luminous source. Indeed, that is just what occurs in any reflection or refraction, which means that the notion of a light 'ray', in so far as optical intensity is concerned, is somewhat problematic here even though Descartes effectively counts rays in his tables. Light rays might be treated as geometrical entities that have a dual character: on the one hand, a ray traces a path of pressure through the balls, while, on the other hand, the number of rays in a given region specifies the degree of pressure sustained by the spheres within it, with the further implication that the pressure acts laterally among the rays as well as linearly along each of them (which follows from Descartes's understanding of pressure).

[59] Despite Descartes' silence on the point, in his *Nova Stereometria* Kepler had explicitly recognized that a property which changes with another will alter the least when the latter is at a maximum (Ch. Frisch, ed., *Joannis Kepleri, Astronomi Opera Omnia* (Frankfurt, 1858–1871), 612, vol. IV), where he writes that 'circa maximam vero utrinque circumstantes decrementa habent initio insensibilia', viz. 'near a maximum the decrements on both sides are in the beginning only imperceptible': translated in Dirk Struik, ed., *A Source Book in Mathematics, 1200–1800* (Cambridge, MA, 1969), 222. I thank Jesper Lützen for the reference. A simple method for determining the extrema of polynomials was developed by Johann Hudde in the 1650s (see Victor J. Katz, *A History of Mathematics. An Introduction*, 2nd edition (Reading, MA, 1998), 473–74).

[60] Johannes Lohne, 'Regenbogen Und Brechzahl', *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften* 49 (1965), 408–409. Since the viewing angle for the primary bow is $4r - 2i$, at an extremum we must have $4dr = 2di$. The law of refraction yields $\cos(i)di = n\cos(r)dr$, with n the index, and so the two relations together entail $\tan(i) = 2\tan(r)$. Barrow (see above, note 51) obtained an equivalent relation, since he had found the conditions which place the intersection of neighbouring refractions on the sphere itself. His version had the form $\cos(r)/\cos(i) = 2/n$: Shapiro, 'The Optical Lectures and the Foundations of the Theory of Optical Imagery', 146, in A.G. Bennett, ed., *Isaac Barrow's Optical Lectures (Lectiones Xviii)* (London, 1987), 146–52.

[61] Johannes Lohne, 'Thomas Harriot Als Mathematiker', *Centaurus* 11 (1965), 35–38.

arithmetically increasing angles of incidence and not for arithmetically spaced incident rays. Using an index for water of 1.335, Harriot computed the viewing angles for the primary bow in increments of 1° of incidence from 56° through 62°. In this region, the differences in the angles of refraction were themselves a nearly constant 0.5°. The viewing angle had the same minimum value at 58°, 59°, and 60°. Lohne tentatively suggested that Harriot could have found the tangent relation from the table by combining the law of refraction with the approximation that the difference between the sines of the successive incidences, and between the sines of the successive refractions, would be proportional, respectively, to the cosines of the incidences multiplied by the 1° difference, and the cosines of the refractions multiplied by the 0.5° difference. Since Descartes's rays, and not his incidences, were spaced arithmetically, he could not have noticed anything like Harriot's relation.[62]

Harriot did not leave any notes concerning the concentration of light near the extrema, although he did refer to the cutoff point as the 'tropical ray'. The astronomical tropics mark the boundaries of the regions within which the Sun can reach the zenith at some point during the year. Perhaps Harriot had something similar in mind, since the extrema mark the boundaries of the illuminated region.[63] Despite Harriot's having actually computed a different index of refraction for the boundary colour at the top of a white surface observed through a prism,[64] he left uncommented the rainbow's tints. Like Descartes at first, Harriot may have been stymied in so doing by the absence of a physical boundary to generate colours in the case of a raindrop.

7. The Apparent Position of the Solar Image and the Order of Colours Within It

Descartes now had to hand not only values for the cutoff angles—the radii of the primary and secondary bows—but also a rationale for the appearance of bright

[62] Descartes tabulated arcs $\widehat{FG}$ and $\widehat{FK}$, which are (180°–) *2i* and *2r*. He accordingly could read off twice the differences between successive angles. The differences in the incidences proper in the vicinity of the primary maximum from distances of 8300 through 8800 (the tabular extremum occurring at 8500 and 8600) are 1° 1′, 1° 2′, 1° 5′, 1° 6′, 1° 9′, 1° 10′; the corresponding differences in the refractions are 1° 3′, 1° 2.5′, 1° 3.5′, 1° 3′, 1° 8′, 1° 8′. The problem is not that Descartes's differences in the refractions are any larger than Harriot's in the same vicinity, but that his incidences are not fixed at arithmetic differences. Consequently, what jumps to the eye in Harriot's table—that the refraction differences are very nearly half the fixed differences in the incidences—cannot be divined from Descartes's.

[63] Alternatively, Harriot may have been thinking of the concentration of light near extrema, since Newton, without having seen anything of Harriot's, made the terminological connection based on the slow motion of the Sun at the solstices: 'Now it is to be observed, that as when the Sun comes to his Tropicks, days increase and decrease but a very little for a great while together; so when by increasing the distance *CD* [which measures the sine of the angle of incidence], these Angles come to their limits, they vary their quantity but very little for some time together, and therefore a far greater number of the rays . . . ' (Newton, *Opticks: Or, a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light. Also Two Treatises of the Species and Magnitude of Curvilinear Figures*, 128: Prop. IX, Prob. IV).

[64] Johannes Lohne, 'Thomas Harriott (1560–1621). The Tycho Brahe of Optics. Preliminary Notice', *Centaurus* 6 (1959), 120. Harriot used both a hollow glass prism filled with turpentine, saltwater, wine or just water, as well as one of solid glass. He found, for water, for example, that what he termed the 'chief primary' light, which must be just below the coloured border, had an index of 1.3351921, while the 'secondary', or red ray, had an index of 1.3415993. Though Harriot as usual left no remarks in MS, his diagram (ibid.) shows that he observed the upper boundary of the paper through the prism apex, where the colour will be red. He did not it seems observe a lower border, which would be coloured blue. Since Harriot gave his measured incidences and refractions to the minute, his computed values for the indexes are accurate to about .002 for angles above 20°.

coloration at them. What he did not as yet have was a reason for the inversion of colours between the two bows. Why should it be that the inner border of the secondary, and the outer border of the primary, are both red (cf. Figure 1)? Why should the colours not appear in both bows in the same order from top to bottom? Descartes did in fact have an explanation for the inversion, one that he first deployed for the order of colours produced by a prism.

We begin by asking, for the prism, what Descartes has not thus far explicitly answered at all: namely, how the side *DF* of the beam that shows red in refraction can be distinguished other than by its colour from the side *EH* that shows blue. What observational criterion sets the two edges apart from one another? We have already seen that the distinction cannot be based upon the relative angles of the beam edges with respect to the interface, and neither did Descartes introduce a second distinction based on whether a beam cross-section drawn from a given edge lies within the medium of incidence or of refraction. But there is a third way, one that is moreover related geometrically to the second. In Descartes's Figure 3, the edges of the beam within the prism that are forged by the aperture *DE* on the exit face are distinguished from one another by the fact that the ray which reaches *E* always has a longer trajectory within the prism than the ray that reaches *D*. Another way to specify which edge has which colour, then, would be somehow to invoke the body of the prism in relation to the ray. At this point in his narrative, Descartes let the matter of distinguishing beam edges lie, but, to jump ahead, he turned explicitly to it after his full discussion of the rainbow's geometry. Because I will offer a particular interpretation of his otherwise puzzling remarks on this critical aspect of the rainbow narrative, it is worth quoting Descartes on it in full. He wrote (referring to Figure 3):

> The same factor, which causes [red] to be near *F* rather than *H*, when it appears through the crystal *MNP*, also brings it about that if we look at this crystal when the eye is in the location of the white screen *FGH*, we will see the red toward its thicker part *MP*, and the blue toward *N*, because the ray tinted red which goes toward *F* comes from *C*, the part of the Sun which is closest to *MP*'.

These remarks make sense if Descartes had the locus of an image in mind, or at least a locus that could be constructed by means of the ancient 'cathetus' method in refraction. That method, according to Johannes Kepler's (1571–1630) renovated optics, had to be generally incorrect, but Descartes did not have a clear understanding of what method to replace it with. The diagrams that he produced in the sixth, seventh and ninth Discourses of his *Dioptrique* for the locations of what he described as perceived images do not seem to be drawn according to Keplerian principles, or perhaps according to any fixed principles at all. Although he knew that the cathetus method did not work in constructing the images perceived in reflection from curvilinear surfaces,[65] he wrote nothing in this regard about refraction, whether at plane surfaces or otherwise.[66] Descartes seems to have considered that image

[65] Descartes remarked 'how much the ancients were deceived in their Catoptrics, when they tried to determine the locations of images in concave and convex mirrors' (AT, v6, 144; PO, 110).

[66] For Kepler, see Shapiro, 'The Optical Lectures and the Foundations of the Theory of Optical Imagery', 119–27, whom I thank for discussions concerning Kepler, Descartes and perceived images. All of the situations with which Descartes was concerned involve sets of divergent rays, and so the 'image' in question is the one produced by the eye or by an equivalent lenticular system. The images are all, in later

distances were obtained by means of two physiological processes: the eye's accommodation in focusing, and the convergence of the two eyes on a given object. Both, he felt, were imprecise and highly limited—'all the means that we have for knowing distance are very uncertain'.[67]

With Descartes, we place 'the eye' at the position of the screen *PHGF* (Figure 12), and we recall that he considered the rays that strike the face *NM* of the prism to enter along the normal. Refraction accordingly takes place only at the exit face *NP*, and so the entire region above *NP* may be considered filled with glass for the purposes of image construction. Looking back towards *DE* from *F*, we will place the apparent image of the light that strikes *F*, which comes from *C*, at I_F at the point on the perpendicular to the interface *NP* from *C* intersected by the projection of the emergent ray *DF*. Similarly, the apparent position of the light which strikes *E* will be I_H. Whatever the respective angles at which the rays from *A* and *C* may strike, the image that forms the red light at *F* will always seem to be closer to *MP* than the image that forms the blue light at *H*.

The method provides a definite construction for the *order of colours* in prismatic refraction. Since the entire structure of Descartes's line of reasoning depended upon his including all necessary, and excluding all unnecessary, conditions of configuration, he could not have expected anyone who had understood his argumentative procedure to accept its cogency in the absence of a determinate method of this sort. Nevertheless, Descartes hardly made the construction limpid, inserting it as he did after considering the rainbow's geometry, and without making it altogether explicit, since he did not explain what it means to say that red at *F* appears to be 'toward' the prism's 'thicker part *MP*'.

My understanding of Descartes's meaning is certainly conjectural, but it does have the virtue of granting his assertion an optical foundation that it otherwise seems to lack. Moreover, we shall see that it can be applied consistently to his remarks on

parlance, virtual—they cannot be received directly on a screen without lenticular transformation. Kepler did not consider the perception of an image by a single eye, which is just the situation that Descartes had in mind for the prism and, subsequently, for the rainbow: viz. (referring to figure 3) 'when the eye is in the location of the white screen *FGH*' (AT, v6, 341; PO, 342).

John Schuster has argued that Descartes may have discovered the law of refraction by reasoning from the cathetus method despite Kepler (John Schuster, 'Descartes and the Scientific Revolution, 1618–1634: An Interpretation', Ph.D., Princeton University, 1977, John Schuster, 'Descartes Opticien: The Construction of the Law of Refraction and the Manufacture of Its Physical Rationales, 1618–1629', *Descartes' Natural Philosophy*, edited by Stephen Gaukroger, John Schuster and John Sutton (London, 2000), 299–329). Schuster's argument is inferential, based in substantial part on similarities between Claude Mydorge's (1585–1647) diagrammatic formulation of the method in a letter to Marin Mersenne (1588–1648) and the way in which the law would be expressed if it did originate in the cathetus. There is substantial evidence, Schuster notes, that both Harriot and Willebrord Snel (1580–1626), both of whom produced the law independently, did reach it in just that way, or at least made the association between image locus and the refraction law. Schuster's essential point is that if an object is placed sequentially on the circumference of a circle within water, for example, then the locus of its image points drawn according to the cathetus will appear to lie on a concentric circle of smaller radius, within observational limits. If the emergent rays are assumed accurately to entail the smaller circle, then the law of refraction follows at once, albeit expressed in a different manner than the ratio of sines.

[67] AT, v6, 144; PO, 110. See also Descartes's remarks in his posthumously published *Traité de l'Homme*, in which he associates the judgement of distance with the displacement of the point in the pineal gland that is moved by the effect of the animal spirits impelled by the visual motions produced, in this case, in the retinas of both eyes (AT, v11, 183). Also see Gary Hatfield, 'Descartes' Physiology and Its Relation to His Psychology', *The Cambridge Companion to Descartes*, edited by John Cottingham (Cambridge, 1992), 357, who remarks Descartes's notion that 'the idea of distance is caused by a brain state without judgmental mediation'.

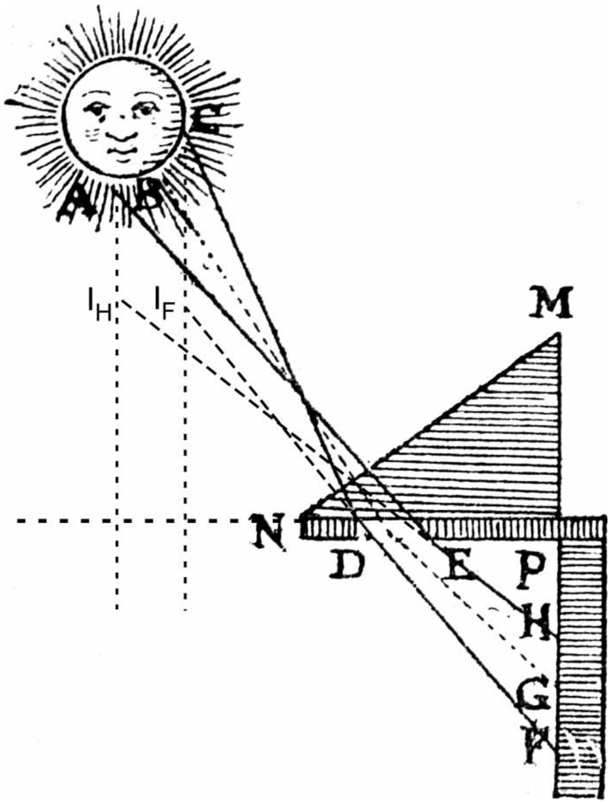

Figure 12. Image loci by the cathetus rule in Descartes's prism.

the order of colours in the rainbow. In fact, the explanation for the prism appears late in his narrative, after he has discussed cutoff angles and ray clustering. The colour-determining criterion for the prism, he wrote there, is the relative proximity of the image produced by the ray to the 'thicker part' of the prism. Descartes continued

> And this same factor also makes it happen that when the centre of the drops of water (and as a result their thickest part) are on the outside, with respect to the coloured points forming the interior rainbow, the red must appear on the outside there; and that when they are on the inside with respect to those which form the exterior rainbow, the red must also appear on the inside.

This is a remarkably obscure explanation, it would seem, for colour inversion, unless we associate it as Descartes did with his rationale for the colour order generated by the prism. The proximity of an image to the prism's 'thicker part' shifts the colour toward red, to the outer part toward blue. In the case of the raindrop, Descartes interpreted the analogous factor to be proximity to the drop's centre—its 'thickest' part. If, accordingly, the factor works for the rainbow as it does for the prism, then the 'images' of the clustered, extreme rays in the two bows should be comparatively close to the drop's centre.

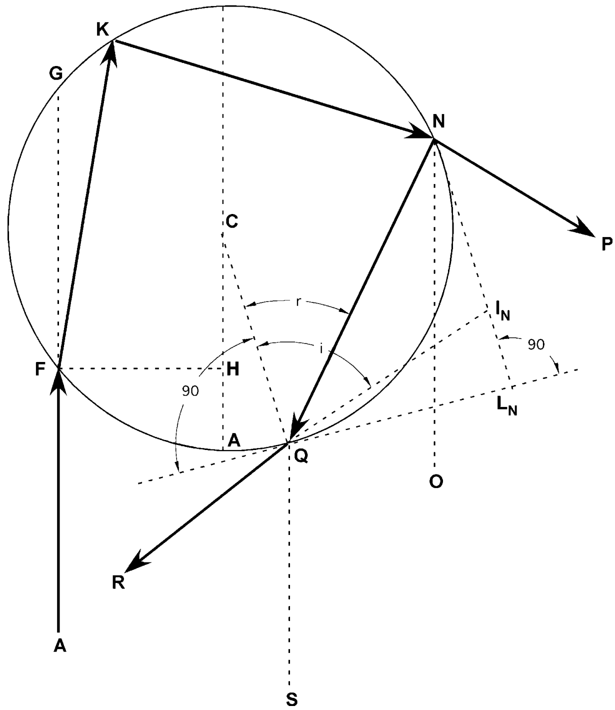

Figure 13. Image locus I_N by the cathetus rule for the twice-reflected light in Descartes's raindrop.

Figure 13 depicts the location of the image I_N for a point of the secondary bow. To construct it according to the cathetus procedure, we draw the tangent at ray *RQ*'s point of emergence (*Q*) from the drop; that ray derives from *QN*, which originates by reflection at *N*, and so we drop a perpendicular from *N* to the tangent drawn at *Q*.[68] According to the cathetus method, the image of *N* will be located at I_N, where *RQ* projected back to the perpendicular from *N* intersects the tangent drawn at *Q*. In the

[68] The sequential application of the cathetus rule to multiple reflections was applied by Hero in his *Catoptrics* to mirrors (Morris R. Cohen and I.E. Drabkin, eds., *A Source Book in Greek Science* (Cambridge, MA, 1975), 267: sec. 18), which (though attributed to Ptolemy) was translated into Latin by the Flemish Dominican William of Moerbeke (1215–1286). In Hero's construction, the locus of the penultimate reflection constitutes the object point for applying the cathetus. In the application that, I suggest, Descartes made to the raindrop, the final action is a refraction, but the penultimate effect is, as in Hero's *Catoptrics*, a reflection, so that the generalization of Hero's procedure to this case would consider the locus of the final reflection to constitute the object point for the emergent refraction.

Medieval opticians do not seem to have concerned themselves with images produced by multiple reflections (much less by a refraction preceded by a reflection) which is hardly surprising since, unlike Hero, they were not interested in temple illusions. Giambattista della Porta (1535–1615) did, however, apply the cathetus to the case of two refractions by a sphere or by a convex lens. In doing so, he did not consider the locus of the first refraction to be the object point for the image produced by the second; he instead retained the locus of the original object for the construction. In an altogether unusual departure from the customary uses of the cathetus, he did not even employ the tangent to the surface at the point of emergence. Instead, he passed a line from the object through the centre of the sphere, placing the image at the point of the latter's intersection with the ray within the sphere (and not the extension of the ray from the eye to the sphere). See Giambattista della Porta, *De Refractione Optices Parte* (Naples, 1593); I thank Sven Dupré for the reference and for discussion about the issue.

figure, the ray *RQ*, which pertains to the secondary bow, and the ray *NP*, which pertains to the primary, both derive from the same incident ray *AF.* Because of the drop's circular symmetry, the images for both *NP* and *RQ* have precisely the same distances from the droplet's centre (CI_N). The droplet's circular symmetry also makes it quite simple to construct the image locus.[69]

Between about .83 and .9 droplet radii's distance from the central line for incoming rays, the 'image' point actually moves into the droplet proper, and this, on Descartes's reasoning, should produce a strong red. The distance minimum is extremely close to the maximum for $\angle ONP$, which means that the primary bow should indeed be strongly red at its upper limit. If the minimum for $\angle SQR$ is obtained at this same point, then we could conclude that the secondary bow should also be quite red at its lower limit. The minima for the image distance and for $\angle SQR$ are in fact somewhat displaced from one another, but by only about a tenth of the droplet's radius. If Descartes had gone this far in probing the application of the cathetus rule—and his use of it remains conjectural—he would in any case know from his reading of Kepler that the construction had to be generally inexact. Other colours would then occur, as indeed they do, for rays that strike closer to the droplet's centre (but whose images are further away), from, say, .8 radii and less. To see all of this, Descartes did not have to compute image distances at all, because his tabulation of the several arcs (Figure 11 right) makes it entirely simple to construct them using a ruler and compass. If we do so using Descartes's tabulated values, then we find that the image is indeed closest to the centre at the single-reflection extremum and that it deviates very little from that position at the extremum for the double reflection.[70] It would have been a matter of moments to check the distances constructively given the tabulation.

Descartes's words concerning the inversion of colours between the two bows are no doubt remarkably confusing, because they might be read as asserting something about a physical inversion of droplets, which is how Carl Boyer construed them,[71] with the predictable result that the argument seems to be insubstantial, to say the least. However, once we translate the claims into image distances, we can understand what Descartes meant when he asserted that the drop centres 'are on the outside with respect to the coloured points' of the primary bow, and 'on the inside' of them for the secondary. The light at each angular position in a bow is produced by drops located along a circular arc specified by the viewing angle. For each viewing angle, there will

[69] The following relations determine the locus of the image via the 'cathetus' construction:

$$CI = FH/n,\ \ FK = 2\sqrt{R^2 - CT^2}$$
$$NL_N = FK\cos(\overset{\frown}{FK}/2),\ \ QL_N = FK\sin(\overset{\frown}{FK}/2)$$
$$I_N L_N = QL_N/\tan(\overset{\frown}{FG}/2),\ \ I_N N = NL_N - I_N L_N$$
$$CI_N = \sqrt{R^2 + I_N N^2 + 2(R)(I_N N)\cos(\overset{\frown}{FG})}$$

[70] Specifically, at the single-reflection extremum (corresponding to an incident ray that strikes at .85 or .86 radii), the image is located at .99 radii from the drop's centre, while at the double-reflection extremum (for an incident ray striking at .95 radii) it is at 1.06 radii, using Descartes's tabulated angles. The image distance from the centre increases very rapidly as the incident ray moves closer in towards the centre from, so that it is entirely reasonable to argue that red should occur near the extrema for both bows.

[71] Boyer, *The Rainbow. From Myth to Mathematics*, 217. Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes*, 224 follows Boyer on this point, though he notes Descartes's remark about prism thickness, while Gaukroger, *Descartes. An Intellectual Biography*, 269 writes that Descartes could not explain the colour inversion.

be a corresponding incident ray, and so a particular image locus at a specific distance from a droplet's centre. In the case of the primary bow, the image distance to droplet centre is a minimum for a drop located at the largest viewing angle, which is where red must appear. Moving to *decreasing* viewing angles shifts into the primary's less-bright but coloured region, eventually reaching blue, with corresponding increases in the image distances to the droplet centres. Consequently, for the primary bow, the centres of the droplets that produce red must be located at a larger viewing angle than the centres of those which produce the other colours. The same holds, or nearly holds, true for the secondary bow, except that the process is inverted, for here as one moves into the bow the viewing angle *increases* along with the image distances to the centre, and the colours accordingly shift toward blue.

8. A Recapitulation of Descartes's Twisty Path to the Rainbow

The path that, I have argued, Descartes's narrative followed in reaching his account of colours and geometry for the rainbow twisted and turned several ways, so it is worth recapitulating its overall course before we turn to contemporary and later reactions. The route to discovery began with an effort to understand the otherwise anomalous 'rosy' violet, which appears only at the border of a sufficiently narrow aperture. It was Descartes's ruminations here, I argued—ruminations over interactions among the little spheres of the imperceptible world—that provided him with a resource with which to generalize the notion of an 'aperture', ultimately changing it from an object that physically blocks light into any cause that alters interactions among the neighbouring spheres, an alteration that Descartes then assimilates to interactions among rays of light, which can be altered by increasing the number of rays within a given region. Descartes's scheme thereby becomes substantially independent of the invisible world in specifying the observable factors that determine both the bow's geometry and the order of its colours. Nevertheless, it is precisely the unseen realm that may have led Descartes, if only obliquely and through an otherwise puzzling tint, to an understanding of how to produce colours in the absence of physical borders, for he built his spheres directly into the tale by placing them squarely between his stories about prism and water-filled globe experiments, on the one hand, and computations of ray paths within a refracting sphere, on the other.

The mechanisms of the Cartesian universe, one might conclude, work together to produce a sort of narrative escalator that carries Descartes ineluctably along, as though it had a logical life of its own. And yet the elaborate details that Descartes presented served only to support his prismatic observations. Even there, the explanatory coherence was weak because Descartes did not specifically offer a method tied directly to mechanical reasoning to distinguish one side of his beam from the other. He turned instead to image distances, though these had nothing to do with mechanism but a great deal to do with a constructive method generalized from an ancient and admittedly inexact rule. Yet here, too, Descartes would have had a defense had anyone queried him on the point (though no one seems to have done so) since the apparent position of an image inevitably raises issues that go beyond the immediate stimulation of the eye to the effect of the animal spirits on the pineal gland. Judgement does not, it seems, enter into the estimation of distance,[72] but the

[72] See above, note 67.

one-to-one correspondence between distance and displacement in the pineal has somehow to be correlated with the motions of the lens in accommodation and/or of the two eyes in converging. That in turn requires some sort of linking principle, which Descartes never specified, leaving open the possibility of deploying the cathetus as an acceptable method in the absence of something better.

Beyond the critical issues of linking colour existence and order to the observable characteristics of the refracting raindrop lie the difficult questions raised by Descartes's understanding of colour in terms of his invisible world. There was first of all the problem, often discussed by historians, of the difference between motion proper and 'tendency' to motion in the absence of a general conception of pressure. A great deal in the Cartesian scheme actually depends upon elaborating a relationship between the translational and rotational motions of his light-transmitting spheres, even if the translation is, as it were, merely a tendency—what a century later one might have termed a 'virtual' translation. I argued that Descartes understood white light as a situation in which this translation couples to rotation in the same way that it does for a ball rolling without slipping. Colours emerge when the link is broken, when the ball as it were both slips and rotates. Or, it would be more accurate, if certainly confusing, to say that colours occur when the ball *actually rotates* but only *tends to translate.*

The rotational speed of a ball when it does not slip accordingly establishes a central criterion for coloration: at this boundary the light is white; once slip occurs, the boundary is traversed, and the ball may rotate more slowly or more rapidly than in the no-slip case, generating colour when it strikes the perceiving eye. If it does so more slowly, then the colour tends towards blue; if more rapidly, then towards red. White, then, constitutes for Descartes a boundary between coloured regions. To shift light to one side or the other of the border requires in the case of refraction what we might call a 'Cartesian aperture', namely a region in which the ray density changes markedly from one place to the next.[73] The Cartesian micro-processes involved in all of this implicate a number of subtle distinctions that Descartes may have fully developed only in his attempts to answer the queries and critiques of Ciermans and Morin, to which I now turn.

9. Ciermans and Morin Critique Descartes

The immediate reactions to Descartes's account of coloration on the part of both Ciermans and Morin concentrated primarily on difficulties that they perceived in his stories about small balls.[74] Descartes had sent three copies of his four treatises to Vopiscus Plempius (1601–1673), professor of medicine at Louvain, who lent Ciermans a copy.[75] Ciermans wrote to Descartes in March, 1638. He was particularly enamored of the *Géométrie*, which he thought should be set out separately and retitled *Mathématiques pures.* The vision of 'a new world in philosophy', which would

[73] Recall that the concentration of rays is one of only two necessary conditions for the production of colour: the other is either the appropriate physical character of a reflecting surface, or else a refraction. Polished mirrors may concentrate rays, but their surfaces are not of the right sort to produce coloration.

[74] The criticisms are also discussed in Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes*, 212–18.

[75] The Latin correspondence with Ciermans is printed in AT, v2, 55–62 and 69–81. The letters are translated into French in Clerselier, ed., *Lettres De Mr Descartes*, 262–72 and 86–303, vol. 1 and (on different pages) in earlier editions.

explain 'all that's hidden in nature by means of sensible and as it were palpable qualities', captured his imagination as well. But the devil lies in the details, and Ciermans spied trouble. He found it in Descartes's prism. He had two initial objections: first, that the rotations which constitute light of different colours coming from different sources would destroy one another, thereby upsetting the entire foundation of the scheme. That objection reflects difficulties in understanding the transmission of overlapping tendencies to motion in fluid media, and Descartes dealt with it in his reply on those grounds.[76] These were not trivial issues in producing converts to a system that relied on physical imagery and intuitions about motions and pressures, but we turn from them to Ciermans' specific objections to Descartes's account of the prism, and *ipso facto* to the rainbow.

Ciermans had doubts about the spheres altogether, since he thought that they would fly from the Sun, thereby depleting it over time, a not unreasonable objection which, however, reflects the highly abridged presentation of the mechanism in the *Dioptrique* and *Météores* as well as the often-voiced confusion over the difference between conatus, or tendency to motion, and motion proper. Descartes's withdrawal of the elaborate *Le Monde* account from publication after Galileo's condemnation foreclosed a fuller comprehension of the scheme for years. Ciermans went further, pointing out that even if the balls were accepted, Descartes's mechanical story nevertheless ran them from air to water, whereas in the prism they run from glass to air. As a consequence, he argued, where Descartes had the most agitation, namely at his *F* (Figure 3), the least should occur and vice versa. Since red appears at *F*, Ciermans concluded, Descartes should associate red with the least agitation and blue with the most, and he saw no reason not to do so.[77]

Ciermans had more, and here we approach the real crux of the matter, because he touched on Descartes's rosy violet, arguing that if red were associated with less, and blue with more, 'agitation', then the rosy tint would occur more naturally than in Descartes's account because at shadow's edge, some spheres would always be retarded. But the shadow's action itself troubled Ciermans, for he could not see why, in Descartes's mechanism, 'shadow was necessary to generate colors'. This struck at the heart of Descartes's complex linkage of the colours of prism and rainbow, because it led to his core point, namely the simulation of a narrow aperture by ray crowding.

Descartes replied to Ciermans on 23 March. After trying to explain why tendencies to motions do not corrupt one another, and explaining as well that the balls are not sent out from luminous bodies, which instead push them, with the pushing force itself being transmitted, Descartes turned to the other points raised by Ciermans. The first, that Descartes had inverted his example by using air to water, he reasonably disposed of by contrasting the effects on the imperceptible balls with those on macroscopic ones. In the experienced world, water resists a sphere's motion through it more than the less dense air, but in the invisible world, the reverse obtains, for there the denser medium affords less resistance. And so a properly illustrative 'comparison' of the two cases should indeed use the less-to-more-dense macroworld to envision what happens in the more-to-less-dense invisible world.

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[76] On which, see Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes*, 213–16.

[77] We will not pursue the point, but Descartes insisted on identifying red with great agitation because he linked greater agitation or tendency also to heat, and hot things are often red or turned red by heat.

Descartes took considerable pains to answer the critical point concerning the role of shadow, and how it works with refraction. His argument depended on symmetry. Refraction alone, absent shadow, cannot produce colour because every sphere would be in the same local condition as its neighbour. Neither can shadow alone produce colour, even though it breaks the symmetry in the neighbourhood of a sphere at the beam's edge, for the following reason. Although the spheres within a beam of white light certainly tend to rotate (at the 'usual' rate), they may do so in randomly distributed directions because the grosser particles of air or water, which can themselves cause differential actions when the spheres encounter them, are indifferently distributed. The actions which take place at beam edges are as a consequence dissipated into a congeries of effects on the spheres within the beam, since no one among them has a preferred direction of rotation, with the result that the tendency of a randomly chosen sphere to rotate remains unaffected overall: the beam stays white as long as the only effect on it occurs at its borders. A refracting surface breaks the dissipation by aligning the rotational directions among the spheres, which permits the shadow borders to act—the symmetry-breaking at the edges continues, but with decreasing effect, from edge to centre. Put perhaps too bluntly, a shadow breaks local symmetry but is neutralized by randomness absent an interface, whereas an interface breaks randomness but is neutralized by local symmetry absent a shadow.[78]

Ciermans had not penetrated the depths of Descartes's scheme either in its manner of ordering colours or in its evolution of the notion of shadow for general application. His queries reflect the difficulties even a careful reader could have. Morin had similar problems, which he had conveyed on 22 February in an extraordinarily long and prolix letter that aimed primarily at issues concerning light.[79] Most of Morin's objections come down to his understandable bewilderment about motions versus tendencies to motion.[80] He was especially perplexed by Descartes's claim that the spheres 'roll in air' when the discussion based on Figure 5 seemed to him to imply that they do so 'only when they encounter a more solid surface'. With all of these problems, Morin asked, 'Monsieur, judge for yourself, according to the first precept of your Method, whether [your explication by means of small balls] must be accepted as true, where there seem to be so many doubts and contradictions'.[81]

Descartes wrote back at nearly equal length on 13 July.[82] He was 'sorry that [Morin] chose to form his objections only on the subject of light' because he had

[78] Descartes of course used neither word in his explanation for Ciermans. Nevertheless, the discussion quite clearly proceeds by compensating one action by another in an unrefracted beam, and by invoking the absence of local differences in an unshadowed one: see Clerselier, ed., *Lettres De Mr Descartes*, 298–303. According to this way of thinking, shadow combined with a sufficient degree of refraction resets an unrefracted beam so that the spheres all now rotate in the same direction (ibid., pp. 300–301) but at different rates. The rate increases from the 'usual' at the centre up to a maximum at the first edge, and decreases from the centre to a minimum at the opposite edge. Without a general ordering of the spheres into the same rotational direction, it would be difficult to understand why the rate is a maximum at one edge and a minimum at the other, since if a random distribution of rotational directions continued to obtain on refraction, then presumably the rate-changing actions would themselves have random effects, leaving the beam overall untinted. However, the direction of rotation still remains irrelevant to colour—only the magnitude of the rotation rate's difference from the 'usual' counts.

[79] AT, v1, 536–57.

[80] 'I would willingly attack the essence or nature of light, which you say is action, or motion, or the inclination to motion, or like an action and a motion, &c. of subtle matter, &c'. (ibid.,547).

[81] Ibid.

[82] AT, v2, 196–221.

'expressly abstained from giving his opinion about it', that is from giving elaborate details, and keeping to his resolution Descartes 'could not perfectly satisfy' him—following which he wrote page after page of reply which nicely avoided satisfaction. He stuck to his resolution, leaving Morin more perplexed than ever. To Morin's specific remark that the balls roll only at their encounter with a 'more solid' interface, Descartes replied with what Morin reasonably took to be a non sequitur:

> I do not speak at all about the subtle matter, but about wood balls, or other visible matter, which are pushed towards water; as is evident from my making them turn completely contrary to the parts of the subtle matter, and compare the rotation which they acquire in leaving air and entering water, to that which the parts of the subtle matter acquire in leaving water or glass and entering air.[83]

Morin's confusion only increased. He answered on 12 August, again at considerable length.[84] Though Descartes had not replied with any remarks about the role of subtle matter, having recurred to wood balls, Morin insisted: 'your own text will condemn you' because he (Descartes) had explicitly written that 'these balls can roll in diverse ways' with no mention of wood.[85] And why bring in wood balls in any case—a remark which in itself shows that Morin had not absorbed the unarticulated subtleties of Descartes's complex reliance on comparisons. Descartes took a month to reply. He explained that he had used 'sensible matter and not small parts of subtle matter' in his explication in order to 'submit my reasons to examination by the senses, as I always try to do'. But this time, Descartes realized that it would be best to explain what he was up to in using macroscopic comparisons, particularly since the method of comparison was in such frequent use among scholastics.[86] He wrote

> It's true that the comparisons which are customarily used in School, explaining intellectual matters by bodily ones, substances by accidents, or at least one quality by another of the same kind, instruct very little; but because in the ones that I use I only compare motions to other motions, or figures to other figures &c., that is to say things which are so minute that they can't be sensed to others which can, and which furthermore don't differ any more from them than a large circle from a small one, I claim that they are the most appropriate way to explain the truth of physical questions that the human mind can have.[87]

Descartes's explanation to Morin was in keeping with his entire approach to microphysical reasoning, which so often used an illustrative macroscopic comparison.[88] The key justification, as he saw it, lay in a comparison's having to be made between things of a like kind. There would certainly be differences in detail between the invisible world and its macroscopic analogue, but the generic character of an effect could illustratively jump the barrier between the visible and the invisible.

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[83] Ibid., 208. Cited and discussed, with a slightly different translation, in Shea, *The Magic of Numbers and Motion. The Scientific Career of René Descartes*, 217.

[84] AT, v2, 288–305.

[85] AT, v6, 331. In fact, Descartes had written 'il faut imaginer', which has an aura of deliberate ambiguity.

[86] AT, v6, 362–73.

[87] Ibid., 368.

[88] See Peter Galison, 'Descartes's Comparison: From the Invisible to the Visible', *Isis* 75 (1984), 323–24 on comparison and the Cartesian imagination.

Morin replied the following month,[89] and this time he saw danger in Descartes's remarks about comparisons. An astrologer, Morin could not find any analogue of the Cartesian type to celestial influence except by way of comparison with the action 'of God himself'.[90] As for the wood balls, Morin interpreted Descartes's 'examination by the senses' to imply a potential experiment, yet one which, he claimed, 'no man in the world' could do—a reasonable point, given what would have to be done to simulate the Cartesian structure. But Descartes did not have experiment in mind: he was thinking rather of visualization based on experience. Morin wanted him instead to stick to the 'subtle matter'. The correspondence ended there, but not because Descartes felt unable to counter Morin's objections. He perhaps recognized that Morin could not be brought along into the Cartesian universe, one in which the behaviour of the invisible world could only be probed by means of macroscopic analogues, but analogues that would always, and necessarily so, be imperfect simulacra of the mechanical circumstances which they illustrated.

10. More Defends and Critiques Descartes While Hooke Criticizes Them Both

A very different kind of critique appeared fifteen years after Descartes's death from the unfortunate effects of life at the Swedish court in winter. In his 1665 *Micrographia*, Robert Hooke (1635–1703) pointed out an apparently devastating problem. Descartes had written that the production of colours requires 'at least one refraction, and even one whose effect is not destroyed by a contrary one'.[91] After quoting Descartes on the point from the Latin edition of the *Météores*, but adding a parenthetical word not in the original,[92] Hooke remarked that this 'principle of his holds true indeed in a prisme where the refracting surfaces are plain, but is contradicted by the ball or cylinder'. There should consequently be no colour at all in the rainbow because every ray emerges from a sphere at precisely its angle of entry.

Descartes's assertion, whether in French or in Latin, however, specified only that colours require that the 'effect' of the first refraction not be 'destroyed' by a subsequent one, which leaves the 'effect' in question open. Hooke took him to mean that the ray on exit must not be refracted through the same angle at which it had originally entered. This can hardly be what Descartes had in mind, because the fact that this is precisely what does occur with a sphere is both obvious and underpins Descartes's own geometry in the *Météores*. He must have meant something else, something that in the case of the sphere does not destroy colours despite the equality among the angles of entry and emergence. We will see in a moment what he likely did have in mind. But Hooke in any case thought that he had more. His principal and, in his view, truly devastating objection,[93] which he called an *experimentum crucis*, was based on a phenomenon unknown to Descartes, in which a beam of light that passes into a thin film or plate, reflects at its bottom, and then passes out again at the top

[89] AT, v6, 408–19.

[90] Ibid., 411.

[91] AT, v6, 330: 'il y en falloit pour le moins une, & mesme une don't l'effect ne fust point destruit par une contraire'.

[92] Robert Hooke, *Micrographia: Or Some Physiological Descriptions of Minute Bodies Made by Magnifying Glasses with Observations and Inquiries Thereupon* (London, 1665), 59: 'quidem talent ut ejus effectus alia contraria (refractione) non destruatur'—'refractione' is absent from the original, cf. AT, v6, 702.

[93] Hooke, *Micrographia: Or Some Physiological Descriptions of Minute Bodies Made by Magnifying Glasses with Observations and Inquiries Thereupon*, 54.

shows colours even though the top and bottom surfaces are parallel to one another. Moreover, as Hooke also pointed out, these colours arise without any 'necessity of a shadow or termination of the bright rays'.

Six years later, the Cambridge Platonist Henry More (1614–1687) published a reply to Hooke's principal critique. More had engaged in a brief correspondence with Descartes just before his death and was quite taken with Cartesian philosophy. He demurred, however, on Descartes's denial of extension to spirit, as well as on the full scope of mechanical explanation.[94] He was urged to write what became his final work of philosophy because, as John Worthington (1618–1671), vice-chancellor of Cambridge, wrote to him in 1667, 'you have as highly commended Des Cartes, as is possible ... whereby you had fired some to the study of [his philosophy that] ... they are enravisht with it, and derive from thence notions of ill consequence to religion'.[95] More's *Enchiridium Metaphysicum* certainly answered Worthington's request, running as it does to a length which, if not extraordinary by seventeenth-century standards, is nevertheless stupefying.

Setting to the side those foolish people who 'betray a too crass and obtuse mind' and addressing himself instead to others of 'a more disciplined and serious judgment', the intrepid More set out to elucidate, to defend and finally to undercut the 'Cartesian hypotheses of light and colours'.[96] In the end, More revived the scholastic notion of intentional species to explain colours, albeit carried now by an 'immaterial' and all-penetrating spiritual substance.[97] Before doing so, however, More explained the Cartesian 'hypotheses' as he understood them and turned explicitly to the critique of that 'most ingenious Micrographer', namely Hooke.

More reproduced Descartes's prism diagram (Figure 3) and asserted, as Descartes had, that red appears at *F* and blue at *H* because 'the globules lying in the shade near the ray *DF* increase the rotation of the globules of the refracted ray *DF*', while the reverse occurs at the other edge. And, like Descartes, More attributed especial power to red, writing of its 'asperity' and 'ferocity'. He then turned to Hooke's two objections, both of which he felt could be countered in essentially the same way. The colours of thin plates as described by Hooke, More asserted, do not involve a situation similar to the passage of light through parallel surfaces because in the former, the light is reflected at the bottom surface and emerges through the top, whereas in the latter, it leaves at the bottom. Although the light emerges in both cases at the same absolute angle at which it originally entered, it has in the case of the thin plate deviated from its original direction at entry, since it emerges on the opposite

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94 There is an extensive body of literature on More. See Alan Gabbey, 'Philosophia Cartesiana Triumphata: Henry More (1646–1671)', *Problems of Cartesianism*, edited by Thomas M. Lennon, John M. Nicholas and John W. Davis (Kingston, 1982), A Rupert Hall, *Henry More and the Scientific Revolution* (Cambridge, 1990) and especially the illuminating introductory discussion in A. Jacob, ed., *Henry More's Manual of Metaphysics. A Translation of the Enchiridium Metaphysicum (1679) with an Introduction and Notes* (Hildesheim, 1995 (1679)).

95 Jacob, ed., *Henry More's Manual of Metaphysics. A Translation of the Enchiridium Metaphysicum (1679) with an Introduction and Notes*, v.

96 Ibid., 148.

97 Ibid., 164–67. More even revived the argument from authority: 'Thus Plotinus, Whose opinion approaches very closely this of ours, and differs in almost no way, except that he is seen to expressly invoke a certain World-Soul and we are concerned to detect in the present case nothing apart from a certain Hylarchic Principle or World-Spirit in general. On which, however, we acknowledge all those sympathies and harmonies of life to be based, and apart from which we divine the phenomena of light and colours would be either not at all or very weak and fading, and not perceptible from almost any distances' (p. 167)—'hylarchic' meaning that More's World-Spirit rules over matter.

side of the normal to the plate's top from the side at which it had entered. This, More in effect argued—and Descartes would likely have replied along the same lines—is sufficient to constitute a novel configuration that permits rotation of Descartes's 'globules' to occur. Hooke's other criticism—that the emergence of a ray from the raindrop occurs at the same absolute angle as its entry—exhibits the same asymmetry: the angle of emergence may be the same, but the ray has been substantially deviated from its original direction. More went on to argue contra-Descartes that 'the motions and rotations of the globules are not made in purely mechanical ways'.

Hooke was not overly taken with More's notion of an incorporeal but space-filling 'hylarchic spirit', against which he offered in 1677 an experiment and remarked that even

> supposing the Doctor had proved there were such an Hylarchick Spirit, what were we the better or the wiser unless we also know how to rule and govern this Spirit? And that we could, like Conjurers, command this Spirit, and set it at work upon whatever we had occasion for it to do.

Indeed, he continued, 'I am yet to learn by what Charm or Incantation I should be able to incite the Spirit to be less or more active, in such proportion as I had occasion for'. More's 'spirit' is even morally disreputable because it 'incourages Ignorance and Superstition by perswading nothing more can be known, and that the Spirit will do what it pleases'. Matter and motion, on the other hand, put regulation and control 'within the power and reach of mans Industry and Invention'.[98]

More, Hooke wrote, 'doth *Canere triumphum ante victoriam*' (cry triumph before victory), for his answers are 'slight and insignificant'. As for the argument against the colours of thin plates, Hooke first of all did not see what mechanical difference deviation from the original direction should make if the angle of emergence is the same as the angle of entry. But beyond that, if More's claim held true, then any reflections whatsoever ought to produce colours, and in particular thick plates should also be coloured, which they are not. Hooke then turned to a description of his own hypothesis concerning colours and wrote nothing further about the rainbow objection, having already rejected More's claim that deviation from the original direction makes a difference.[99]

Descartes's theory of colours was discussed for many years, particularly as his philosophy gained ground. Hooke's alternative continued to rely on the notion that

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[98] Robert Hooke, 'Lampas: Or, Descriptions of Some Mechanical Improvements or Lamps & Waterpoises. Together with Some Other Physical and Mechanical Discoveries', *Lectiones Cutlerianae, or a Collection of Lectures Physical, Mechanical, Geographical, & Astronomical. Made before the Royal Society on Several Occasions at Gresham Colledge. To Which Are Added Divers Miscellaneous Discourses.* (London, 1679 (1677)), 33–34 in Lampas.

[99] Hooke might have noted a correlated criticism for the rainbow: namely, that if colours *do not* occur when light emerges from thick plates after an internal reflection, then why should they do so in the case of a sphere, which apparently satisfies precisely the same conditions? More's argument based on deviation cannot answer this objection, which is moreover one that Descartes might have known since he was well aware of internal reflection—after all, it underlies his theory of the rainbow. The only way to solve the problem, and one which Descartes would likely have used, involves taking account of the shape of the emergent beam, for there is a significant difference in this respect between *unshadowed* light directly from the Sun that emerges after internal reflection from a plate and the same light that emerges from a sphere: the light from the plate remains effectively without shadow, whereas a marked shadow has been produced in the case of the sphere, thereby providing a rationale for colour. Suppose, however, that a wide beam of

colours are effects produced at the boundary between the lit and the unlit, and moreover that the red and the blue are distinguished among one another by which edge of his moving front first strikes the refracting interface—in this, he had the advantage over Descartes, who resorted to other criteria to specify what colour is generated where. Newton's own alternative did gradually make headway, particularly in Britain,[100] but vestiges of the notion that colours are border phenomena remained for many years, appearing full-blown once again in Goethe's *Farbenlehre*.

11. Descartes Redivivus

We have accompanied Descartes on a long journey from the prism through his invisible world by way of experiment and geometry to arrive finally at the rainbow. My argument has about it something like Dorothy's voyage to the land of Oz. We begin with the observable world, the world of the rainbow itself. Suddenly, a prism transports us through a liminal realm to the strange and busy Cartesian microcosm. We wander for a while throughout an unfamiliar landscape, trying to avoid various traps while looking for ways to take us home. Eventually, we find our way out by means of a new machine forged by the marriage of experiment with geometry. This brings us at long last back home, to the rainbow world, equipped with a new understanding of what it is really all about.

It is perhaps a second-hand sort of Descartes that I have created here, one whose very existence can be divined only indirectly. His presence is signalled by gaps in the discovery narrative that Descartes contrived to persuade readers that the Cartesian rainbow reproduced the natural wonder. To uncover what lies behind these ruptures, I reproduced Descartes's experiments. We then followed his occasionally fractured reasoning through the invisible world in light of what the experiments indicated to Cartesian eyes. And so we were able to uncover factors that provided the intrepid, if overly subtle, Descartes with the resources to explain more than the rainbow's geometry, namely how, and according to what rules, it produces colour.

Discovery narratives are almost always difficult to interpret in the absence of further evidence, and hard enough even then.[101] But it does take a master of rhetoric with an utterly clear view to reconfigure events that he may only weakly remember in order to forge a thoroughly coherent and persuasive story. Luckily for the historian,

shadowed light enters both plate and sphere: it too will emerge uncoloured from the plate but not from the sphere, even though the shadow exists in both cases before entry. To handle this situation in Descartes's scheme would require specifying that the shadow must be significantly altered in the exit beam from its structure on entry, thereby (in effect) generating a new and possibly colour-producing situation. There is indeed a difference available here: in the case of the plate, the shape of a beam is unchanged at emergence; the sphere, on the other hand, markedly alters it, splaying the beam out and redistributing the rays within it. In fact, the rays that had been shadowed at entry are generally not shadowed at exit, and the rays that on exit are now shadowed were not so on entry. All of which offers ample opportunities for exploitation. Hooke's thin films certainly counter this, but the point is that in their absence, Descartes's scheme, at least in this respect, could be reasonably well defended.

[100] Alan E. Shapiro, 'The Gradual Acceptance of Newton's Theory of Light and Color, 1672–1727', *Perspectives on Science* 4 (1996) 59–140.

[101] For another example of imperfect rhetoric, but one which in this case has the virtue of being backed by manuscript evidence, see Jed Z. Buchwald, 'The Scholar's Seeing Eye', *Reworking the Bench: Research Notebooks in the History of Science*, edited by Larry Holmes, Jürgen Renn and Hans-Jörg Rheinberger, Archimedes (Dordrecht, 2003) 309–25. And for an account of how to recover past practice by a master of science history, see Frederic L. Holmes, *Investigative Pathways. Patterns and Stages in the Careers of Experimental Scientists* (New Haven, CT, 2004).

Descartes's rainbow narrative hardly constitutes a paragon example of rhetorical perfection. Luckily, too, he told enough about his work with prisms, as well as his computational procedures, to make possible a recovery of what he perceived. To do so, however, due regard must be paid to the fact that both the laboratory and the links that could join observation and manipulation to calculation were only just being explored in the first half of the seventeenth century, not least by Descartes himself.

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