Chapter 22
Cauchy's Theory of Dispersion Anticipated by Fresnel

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In 1836 Augustin-Louis Cauchy (1789–1857), having left Paris and settled in Prague following the July Revolution, published a memoir on the dispersion of light under the auspices of Prague's Royal Society of Sciences.[1] In it he produced an equation that is even today known as Cauchy's formula for dispersion. It works reasonably well for normally dispersive bodies and was only replaced towards the end of the 19th century following the discovery of anomalous dispersion in Denmark by C. Christiansen in 1870 and consequent changes in theory by Wolfgang Sellmeier and Hermann von Helmholtz (1821–1894) in Germany.[2] In his publication Cauchy nowhere referred for inspiration to Augustin-Jean Fresnel (1788–1827), the originator in France of wave optics. Instead, he wrote that Gustave-Gaspard Coriolis (1792–1843), having read Cauchy's earlier work on the equations of motion that govern a system of material points, suggested that terms which Cauchy had there neglected might account for dispersion – assuming that the medium, or ether, that was presumed to carry optical radiation is itself so constituted.[3]

Cauchy's effort in optics was preceded by his major innovations in elasticity theory, which he was stimulated to investigate when he read a paper by Claude-Louis Navier (1785–1836) on the theory of elastic plates. Navier had submitted the paper to the *Académie des Sciences* in Paris on August 14, 1820 and had given lithographic copies to a number of academicians, including Cauchy. An abstract was printed but only in 1823. However, in 1821 Navier developed a full theory of elasticity based on a consideration of forces between particles, which

[1] Cauchy, A.-L., 1836; Cauchy, A.-L., 1836 (1895). The memoir had originally appeared the previous year, published as a separate installment of Cauchy's ongoing series, the *Exercises de Mathématiques* (Cauchy, A.-L., 1835).

[2] On which see Buchwald, J. Z. (1985, pp. 233–37).

[3] Cauchy, A.-L., (1836, ~~p. 1 1836~~ (1895), p. 196).

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he read to the *Académie* on May 14 but which, like his paper of 1820, remained out of print and under evaluation. This major work on elasticity was not published until 1827.[4]

Cauchy informed the *Académie* of his own results concerning elasticity on September 30, 1822, though he neither read his paper at a weekly meeting of the organization nor did he deposit a manuscript with it, as was customary. That paper nevertheless contained his path-breaking introduction of the concept of stress, there introduced without any consideration of the physical structure of the elastic body, which he treated as continuous. Cauchy contributed a summary account of his results in the *Journal de la Société Philomatique* four months later in its January issue, wherein he did note that he had undertaken his research "on the occasion of a memoir published [publié] by Navier on August 14, 1820."[5] At that time Navier's paper had of course not been printed in a journal, just lithographically copied, though Cauchy's "publié" no doubt simply meant "made known." The abstract of Navier's 1820 paper on elastic plates in fact appeared in the same journal pages after Cauchy's.

In the summary account, Cauchy recalled having spoken with Fresnel. He had just developed his own approach to elasticity when ". . . M. Fresnel came to talk with me about some investigations on light that, as yet, he had only presented in part to the Institut. I learned that he had obtained a theorem analogous to my own, his result being based on certain laws according to which elasticity emanating from a single given point varies in different directions."[6] The "theorem" to which Cauchy referred derived ultimately from Fresnel's efforts to deduce what he termed an optical "surface of elasticity," whose properties he succeeded in developing shortly before March 1822. The problem that had given rise to his search for such a thing concerned the behavior of light in crystals. To solve this required a generalized form of the wave front, and that, he knew, would be a complicated matter to deduce.[7] Fresnel accordingly needed a reasonably straightforward way to reach it, hence the much simpler elasticity surface, which has the form of an ellipsoid with three unequal axes. It has special sectioning properties from which the intricate wave surface could ultimately be found (though even here Fresnel had to take a shortcut), and he justified these properties by invoking the physical characteristics of the ether.[8] As in contemporary French understanding of material elastica, Fresnel's

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[4] Navier, C.-L. (1827).

[5] Cauchy, A.-L., 1823 (1958).

[6] Translated in Belhoste, B., 1991, p. 94 from Cauchy, A.-L., 1823 (1958), p. 301. Belhoste details the events surrounding Navier's and then Cauchy's presentations.

[7] The general wave surface is indeed complicated, satisfying as it does the following equation for a surface of two sheets: $\frac{a^2x^2}{r^2-a^2}+\frac{b^2y^2}{r^2-b^2}+\frac{c^2z^2}{r^2-c^2}=0$ where *a,b,c* are constants pertaining to the specific biaxial crystal. If two of the constants are equal to one another, say *b, c* then this reduces to the wave surface in a uniaxial crystal (viz a sphere and an ellipsoid), which was the only type known until the 1810s.

[8] Buchwald, J. Z. (1989b, pp. 260–90).

optical ether was thought to consist of interacting particles. Navier had in fact used just that model in his 1820 paper for material bodies, as well as in the later paper of 1821, though in 1820 he had not developed it in detail – with the result that a major controversy with Siméon-Denis Poisson (1781–1840) was to erupt over the subject in later years.

After considerable efforts beset by initially deceptive paths, Fresnel in due course succeeded in constructing an empirically workable "surface of elasticity," one which has the necessary property for his purposes that a displacement along any one of its semiaxes gives rise to a parallel restoring force. He was able to deduce the surface from the proposition that the force generated in reaction to a displacement is a linear function with constant coefficients of the displacement's components along three mutually-orthogonal directions. That is, the relation between reaction and its generating displacement involves what in modern terms is a linear transformation with constant coefficients, whose symmetry Fresnel demonstrated on the basis of a balance of moments. Fresnel did not however by his own admission provide a physically-acceptable foundation for that critical proposition since he had found it necessary to make the obviously unphysical assumption that the reaction to an ether particle's displacement could be calculated by shifting it alone, leaving all the others in situ.[9]

That linear transformation was the specific result to which Cauchy referred in his 1823 remark, though his form of it for material bodies emerged, unlike Fresnel's for the optical ether, from general considerations of symmetry. His version specified that the directed force on a given plane subject to elastic deformation can be found from a linear transformation applied to the plane's normal. In effect, Fresnel's ether displacement stood in the same relation to the force that is associated with it as Cauchy's normal to a given plane within a deformed elastic body stood to the corresponding force on that plane. Cauchy accordingly recognized that his transformation was the same in essence as Fresnel's, and that it led to what has since been termed the stress quadric – which is also Fresnel's "surface of elasticity."[10] These results eventually enabled Cauchy to develop general equations of motion for elasticity without relying on any particular physical model, e.g. of material points.

We do not know precisely what Fresnel and Cauchy discussed in their meeting, which must have taken place between late 1821 and the early fall of 1822. He left vague just how far their colloquy had extended, for he mentioned only that Fresnel had obtained a "theorem analogous to my own," viz the linear transformation in question, as well as the associated surface. He did not write anything about Fresnel's foundation of the "theorem" in particle interactions (faulty though that foundation was even to Fresnel). However, it seems quite

[9] The sequence of Fresnel's investigations is complex: see Buchwald, J. Z. (1989b, pp. 260–90) for details. Fresnel's final surface of elasticity has the equation $r^4 = a^2x^2 + b^2y^2 + c^2z^2$.

[10] He remarked in a note that from the 'theorem' in question there resulted a surface with properties that "agree with the final researches of Fresnel."

likely that, until then, Cauchy had not considered the ways in which his new conception of stress could be derived from, or at least connected to, the kind of model that Fresnel had deployed, and with which Navier had also worked in the paper that he had read to the *Académie* in 1821. Still, Cauchy did note that they had discussed Fresnel's "investigations on light," and it is entirely possible – indeed, we shall see quite likely – that their discussions ranged over more than the issues raised by the transformation, the stress quadric, and its consequences for developing the several surfaces needed for Fresnel's final theory of birefringence. In any case, it seems reasonably certain that Cauchy's further developments were at least stimulated by his discussion with Fresnel.

The detailed presentation of Cauchy's new theory of stress appeared in print five years after his meeting with Fresnel, in the second volume of his *Exercises de Mathématiques* (1827). An addendum to the piece presented a deduction of the symmetric transformation that is implied by the three general theorems which Cauchy developed in the body of the article. That deduction relied for the first time on the basic elements of the particle-based model that Fresnel (and Navier) had deployed. Here Cauchy noted that the transformation and "many propositions which can be deduced from it and which are analogous to the theorems I, II, III" of his own presentation were "due to Fresnel." He continued to leave open the question of whether he, Cauchy, had also learned anything from Fresnel with respect to the underlying deductive structure whose elements he outlined.[11]

Years before, in fact not many months after their meeting in 1822, a tense situation had developed between Cauchy and Fresnel when Cauchy preempted Navier by publishing his 1823 summary in the *Philomatique*. This had generated an angry reaction on Navier's part that eventually drew in Fresnel. Neither of Navier's memoirs had been printed or even formally evaluated by the *Académie*, and Cauchy's summary only gave backhanded acknowledgment to Navier's less-general paper on plates as marking the moment at which his own research had begun. In fact, the entire first paragraph of the summary was devoted to dismissing Navier's geometrical distinction between forces of flexure and forces of dilatation, replacing both with a unified concept of stress. Cauchy, recall, had told the *Académie* about his results the previous September 30. That alone had been enough to distress Navier, who wrote the *Académie* on October 6 asking to have his papers rapidly evaluated, remarking without comment that Cauchy was working a similar vein. Matters rapidly turned worse with the *Philomatique* publication, and in March Navier spoke of his own papers before that society, with an "extract" of his research on elastic plates and a second concerning his general theory appearing in the society's journal. Support turned to Navier, and Fresnel jumped in with a strong, public note criticizing Cauchy.[12]

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[11] See equation 12, Cauchy, A.-L., 1827 (1889), p. 81.

[12] Belhoste, B. (1991, pp. 97–8): *cf* Fresnel, A. (1823) and Navier, C.-L. (1823a); Navier, C.-L. (1823b).

Fresnel's critique primarily concerned Navier's second memoir (his general theory of elasticity). Cauchy's summary seemed to be quite similar to that work, Fresnel remarked, work that had been talked about before the members of the *Philomatique*. "It is important," Fresnel went on, "that the date of this paper [Navier's second, general one] be recalled and certified."[13] The tone of Fresnel's remarks clearly shows, as Belhoste notes, that he believed Cauchy had likely taken results from Navier. At the time Fresnel thought that Cauchy was one of the examiners assigned by the *Académie* to evaluate Navier's memoir, though in this he was mistaken since the official examiners were Gaspard de Prony (1755–1839), Poisson, and Jean-Baptiste Fourier (1768–1830). Fourier himself entered the fray on April 24 when, in reporting on the *Académie*'s events from 1822, he noted that Cauchy had presented a paper on September 30 in which he cited Navier and Fresnel as having "already treated questions of the same kind." He went on explicitly to note that Navier had given two papers, and that Fresnel's optical work had led Fresnel to examine "the properties of vibratory motions that occur within the interior of elastic bodies."[14]

Fresnel's sharp note critiquing Cauchy may have reflected his own experiences at the time with respect to publication. In the early summer of 1821, a confrontation over memoirs by Fresnel had taken place at the *Académie*. François Arago (1786–1853) had written a long-delayed report supporting Fresnel's work on chromatic polarization (in which colors appear when white light is passed through thin crystal slices). There he directly attacked the earlier, lengthy theories of the phenomenon by Jean Baptiste Biot (1774–1862), who continued to think that light consisted of independent rays that mark the paths of optical particles. Stung by the critique from someone with whom he had long had conflicts, Biot accused Arago of having intentionally delayed the report. Fresnel wrote his brother about the events on June 13.

It's entirely clear from the two original papers that Fresnel had written, and on which Arago reported (neither of which was printed in their original form during Fresnel's lifetime), that Fresnel himself had not targeted Biot, likely preferring to keep clear of potential difficulties. These events would no doubt have made him particularly alert to the appearance of remarks about papers still under examination. Moreover the tone of his letters, both then and earlier, clearly indicates that Fresnel was sensitive to issues of priority in discovery, so that Cauchy's having pushed Navier to the side and having barely mentioned Fresnel in his summary report likely rankled.[15] That in turn suggests Fresnel may have discussed a good deal more than Cauchy mentioned either in 1823 or in 1827 (in the latter case, after Fresnel's death).

It may be that Cauchy decided not to fully publish his investigations on elasticity at the time, as Belhoste has suggested, in part because of this

13 Cited and translated in Belhoste, B. (1991, p. 98).

14 Fourier, J.-B. (1823, p. 258).

15 On these events see Buchwald, J. Z. (1989a, 1989b, pp. 237–51).

contretemps, waiting until Navier's own paper reached print in 1827.[16] That would have the further advantage of allowing him time to develop a physical foundation for an elasticity theory based on particle interactions, thereby completing his marginalizing of Navier. In fact, Belhoste points out, Cauchy claimed in a published note in the *Philomatique* that even his first memoir, on elastic plates, had been based on particle interactions, though it certainly had not been.[17] Then, on October 1, 1827, Poisson announced that he was working on a "far-reaching study" of elasticity, and this led Cauchy the same day to deposit his own memoir as a *pli cacheté* with the *Académie* to establish priority.[18] Further arguments with Navier and Poisson followed which we need not consider here, because Fresnel had died on July 14 in Ville-d'Avary "in the arms of his mother."[19]

On June 7 and 14, 1830, three years after Fresnel's death, Cauchy presented a comparatively short (given his customary standards) memoir on light before the *Académie* in Paris, which appeared thereafter in the *Bulletin de Férussac*; he also had it printed separately by de Bure Frères – the latter being publishers to the king, among others.[20] He then left the city in the first week of September, probably not at first intending to go into exile but rather to rest after the, to him, dispiriting events of the July revolution and following his exhaustingly extensive record of publication and presentation of memoirs.[21] His absence turned into a true exile, taking Cauchy at first to Fribourg, then Turin, and eventually to Prague in 1833 as tutor in the sciences to the exiled monarch's notably recalcitrant son, the Duke of Bordeaux. Then, in 1835, Cauchy published there an extensive memoir on light that was based on equations he had developed for a system of interacting particles. This memoir contained the expression for optical dispersion that was reprinted the next year under the auspices of Prague's Academy and that continues to appear under his name to this day.

Two years before leaving Paris, Cauchy had written three papers on these particle equations in an attempt to provide a physical basis for his general theory of elasticity. They had appeared in his on-going *Éxercises de Mathématique*. The following year he published a paper in the *Bulletin de Férussac* that

[16] Belhoste, B. (1991, pp. 98–99), who remarks that during the intervening years Cauchy modified his continuum theory to incorporate two elasticity constants, thereby making Navier's a special case since its reliance on particle interactions produced a single constant. See Darrigol, O. (2005, pp. 109–25); Grattan-Guinness, I. (1990, pp. 968–1045) for accounts of developments in elasticity theory at the time.

[17] Belhoste, B. (1991, p. 99).

[18] Belhoste, B. (1991, pp. 99–100).

[19] Verdet, E. (1866, p. xcviii).

[20] Cauchy, A.-L. (1830a, 1830b, 1830 (1958)-a). The *Bulletin* was founded in 1823 and continued through 1831 in part with the aim of ensuring rapid publication by young scholars who may not have been held in high regard by the leaders of their fields, as well as by known experts (Taton, R., 1947). Cauchy published papers there in 1828 and 1829.

[21] As suggested in Belhoste, B. (1991, pp. 145–6).

for the first time briefly described applying the results to optics.[22] Then, after the appearance in 1830 of the memoir on light that he had presented on June 14 of that year before the *Académie*, Cauchy published one further paper on light (in *Férussac*, on which more shortly), and nothing more until 1835.[23] We do however know that at the time of his June 14 presentation he also announced to the *Académie* that he "had the formulas relative to the *dispersion of light* that he had read at the last session." The *Procès Verbaux* for the meeting accordingly noted that Cauchy had presented a memoir "on the subject."[24]

Perhaps Cauchy waited until after Fresnel died to move ahead in public with a theory of elasticity based on particles because of the latter's angry note in 1823. That would certainly have avoided any reactions from the departed Fresnel. The question we must now pursue is whether the key new result of that work for optics – Cauchy's formula for dispersion, which reached print for the first time in 1835 – was wholly original in concept and form with him, for there exists an unpublished manuscript by Fresnel dated July, 1822 in which he sketched a theory of, and produced a formula for, dispersion near the very time that we know Fresnel met with Cauchy to show him some results about light.

Cauchy's first published remarks on dispersion had appeared in his second 1830 paper on light, which concerned reflection and refraction.[25] That paper referred back to the particle equations that he had developed over the previous two years and that had been printed in 1828 and 1829. In those papers Cauchy had reached the general equation (22.1) below for the ether (or for that matter for any material body similarly constituted of interacting particles). At that time he had continued by expanding the differences in the particle displacements ($\delta^i\mathbf{u}$) in Taylor series about the particles' equilibrium loci. He had then dropped terms beyond second order and imposed isotropy on the system, thereby obtaining the following equation of motion, which reads in modern notation[26]:

$$m\frac{\partial^2\mathbf{u}}{\partial t^2} = (R+G)\nabla^2\mathbf{u} + 2R\nabla(\nabla \bullet \mathbf{u})$$

Here R and G compact the constants of the isotropic system. Cauchy noted that if he had retained the expansion through the fourth order, then terms in $\nabla^4\mathbf{u}$ would appear, and that these would produce dispersive effects, i.e. that the wave speed would then depend upon the wavelength. This much was entirely obvious,

[22] Cauchy, A.-L. 1828 (1890)-a; Cauchy, A.-L., 1828 (1890)-b; Cauchy, A.-L., 1828 (1891); Cauchy, A.-L., 1829 (1958).

[23] Cauchy, A.-L., 1830 (1958)-b.

[24] Anonymous, 1830.

[25] Cauchy, A.-L., 1830 (1958)-b, pp. 155–57.

[26] In the case of Cauchy's work in this area, transforming his equations into vector form does make it much simpler to grasp their structure, but it also traduces to a certain extent the difficulties he faced in forging the system out of a morass of algebraic relations with often perplexing geometric connections.

and not only to Cauchy. The question was how to develop the system's equations to yield a proper formula. At the end of his paper Cauchy remarked that he had described how to do just that in his lectures on June 19 and 22 at the Collège de France, and that he would explain it in "more detail in a new article." That article seems never to have appeared, most likely because of the chaotic events surrounding his departure from Paris. In any case, he may have had in hand many of the results that first reached print in 1835.

The essentials of the structure that Cauchy had begun to develop after Fresnel's death and that he used to produce a dispersion formula are deceptively simple. Imagine an arrangement of point like particles each of which acts on all of the others with a repulsive force. We do not initially make any assumptions about the arrangements of the particles, in particular what symmetries the system might obey, and neither do we specify the form of the force other than to assume that it falls off with distance. Through their interactions these particles establish a pattern that results in wave propagation, and Cauchy aimed rigorously to analyze the system in order to generate optical equations.

Each particle in the system acts to produce an acceleration $f(r)$ on every other one that is directed along the line joining each pair, that depends on their masses and on their mutual distance r, and that, like gravity, satisfies Newton's third law. The system has an equilibrium configuration in which the net force on every particle vanishes, thereby providing Cauchy with a first condition that the constants of the system must satisfy, namely that the following relation for the force on any given particle of mass m must hold:

$$m \sum_i m_i f(r^i) \mathbf{e}_{r^i} = 0$$

Here the r^i represent the distances in equilibrium between the given and the ith particles, $\mathbf{e}_{r^i}$ is a unit vector along $\mathbf{r}^i$, and m, m_i are their respective masses.

If, next, our given particle m is displaced by an amount $\mathbf{u}$, then it will experience a net force. The other elements of the system are also assumed to be shifted from their equilibrium loci (this was the admitted defect in Fresnel's deduction of his optical "surface of elasticity"), as a result of which the distance $\mathbf{r}^i$ changes to $\mathbf{r}^i + \delta^i\mathbf{u}$, wherein $\delta^i\mathbf{u}$ accordingly represents the directed difference between the displacements from equilibrium of m, m^i. The fundamental equation of motion is then:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \sum_i m^i f(\mathbf{r}^i + \delta^i \mathbf{u}) \mathbf{e}_{r^i + \delta^i \mathbf{u}}$$

In consistency with the equilibrium condition, $\mathbf{e}_{\mathbf{r}^i + \delta^i \mathbf{u}}$ is a unit vector along $\mathbf{r}^i + \delta^i\mathbf{u}$. The acceleration $f(\mathbf{r}^i + \delta^i\mathbf{u})$ can be expressed in terms of $\mathbf{r}^i$ and $\delta^i\mathbf{u}$ together with the equilibrium values $f(r^i)$ and the latter's derivatives with respect to the r^i by series expansion. If, further, the displacements of the particles are all "small" (meaning through distances much less than the presumed average distance between neighboring particles at any time), then, Cauchy assumed, quadratic terms in $\delta^i\mathbf{u}$ may be neglected in the expansion. Taking into account the equilibrium condition proper, the fundamental equation thereby becomes[27]:

$$\frac{\partial^2\mathbf{u}}{\partial t^2} = \sum_i m^i \left[\frac{f(r^i)}{r^i}\delta^i\mathbf{u} + \left(r^i\frac{\partial f(r^i)}{\partial r^i} - f(r^i)\right)\left(\frac{\mathbf{e}_{r^i}\bullet\delta^i\mathbf{u}}{r^i}\right)\mathbf{e}_{r^i}\right] \tag{22.1}$$

Cauchy over time rang several changes on the results that he could draw from (22.1), altering for example his interpretation of how to express the $\delta^i\mathbf{u}$ as functions of the equilibrium distances r^i. In all cases he recurred to what are, in modern terms, eigenvalue techniques in order to obtain expressions for propagation.[28] He was in this way able to obtain a reasonably close approximation of Fresnel's wave surface for biaxial crystals, the crowning glory of the latter's optical theory.[29] And Cauchy obtained as well what he implicitly claimed to be his own expression for dispersion, one that soon generated considerable discussion, particularly in England.

In order to reach tractable equations that would lead to a dispersion formula, Cauchy first imposed central symmetry on his system of particles, followed by complete isotropy.[30] In an extraordinarily intricate analysis running to many pages and equations, he thereby demonstrated that Ω^2, the (squared) rate of propagation for a given wavelength in the medium, can be expressed as an infinite series in the reciprocals of the wavelength's even powers:

$$\Omega^2 = b_o + \sum_{p=1} \frac{b_p}{\lambda^{2p}} \tag{22.2}$$

[27] This equation occurs first in Cauchy, A.-L., 1828 (1890)-b, pp. 227–31 and then in the major dispersion memoirs (Cauchy, A.-L., 1836, pp. 1–5, 1836 (1895), pp. 195–200). It is briefly discussed in Buchwald, J. Z. (1979, p. 251) as well as in Dalmedico, A. D. (1992, pp. 347–50) but is misprinted in both places. Cauchy likely had most of these in hand by 1827 since he had produced formulae for the force on a surface in such a system that (this being Cauchy's main point at the time) are equivalent in form to his symmetric matrix for the continuum case, formulae that contain sums similar to those for the terms in equation (22.3) below (see Cauchy, A.-L., 1827 (1889), p. 81).

[28] For details on this and reactions in England to Cauchy's dispersion theory see Buchwald, J. Z. (1979, pp. 252–56).

[29] On which see Dalmedico, A. D. (1992, pp. 351–76). Along the way Cauchy felt it necessary to change his views concerning the relationship between the displacement vector and the optical plane of polarization.

[30] In central symmetry, if a particle lies on an arbitrary line through any given particle, then a corresponding particle must also lie on the other side of the line at an equal distance from the given particle.

The particle distances in equilibrium as well as the forces involved are packed into the values of the *b* constants, and so we must next investigate how these constants emerge from the fundamental physics of the situation. That, in turn, will enable us to probe any connections between Fresnel's work on dispersion and this theory of Cauchy's.

We begin with the manner in which Cauchy solved his fundamental equation (22.1). He assumed first of all that its solutions $\mathbf{u}$ could be represented as $\sum_l \mathbf{c}_l e^{i\mathbf{k}\bullet\mathbf{r}}$ wherein the time dependence is assigned to the numbers $\mathbf{c}_l$, taken as functions of position $\mathbf{r}$ and time, and with the $\mathbf{k}$ being real. The latter points in the direction of a disturbance's propagation and is orthogonal to the corresponding (plane) front. Cauchy tacitly took the position vector $\mathbf{r}$ to be effectively the same as the equilibrium loci $\mathbf{r}^i$ of his particles. Taking a single term in the solution series, he then manipulated it into a form containing $\mathbf{u}$, $\mathbf{k}$, and $\mathbf{r}$ which he then inserted into (22.1). Assuming central symmetry, Cauchy could then rewrite (22.1) in the following way:

$$\frac{\partial^2\mathbf{u}}{\partial t^2} = -\begin{pmatrix} L_x & P_{xy} & P_{xz} \\ P_{xy} & L_y & P_{yz} \\ P_{xz} & P_{yz} & L_z \end{pmatrix}\mathbf{u} \tag{22.3}$$

with the matrix elements having the following forms. In these expressions corresponding subscripts are taken by position in each of the six *L,P*:

$$L_{x,y,z} \equiv 2\sum_i \left\{ m_i \left[\frac{f(r^i)}{r^i} + \frac{\left(r^i \frac{\partial f(r^i)}{\partial r^i} - f(r^i)\right)}{r^i} \left(\frac{\left|\mathbf{r}^i_{x,y,z}\right|^2}{(r^i)^2} \right) \right] \sin^2\left(\frac{\mathbf{k}\bullet\mathbf{r}^i}{2}\right) \right\}$$

$$P_{xy,xz,yz} \equiv 2\sum_i \left\{ m_i \left[\frac{\left(r^i \frac{\partial f(r^i)}{\partial r^i} - f(r^i)\right)}{r^i} \left(\frac{\left(\left|\mathbf{r}^i_{y,z,x}\right|\left|\mathbf{r}^i_{z,x,y}\right|\right)^2}{(r^i)^2} \right) \right] \sin^2\left(\frac{\mathbf{k}\bullet\mathbf{r}^i}{2}\right) \right\}$$

These could in turn be made more compact by introducing two functions *U* and *V*:

$$L_{x,y,z} = U + \frac{\partial^2 V}{\partial x^2, y^2, z^2}$$

$$P_{xy,xz,yz} = \frac{\partial^2 V}{\partial yz, zx, xy}$$

in which

$$U \equiv \sum_i m_i \left\{ \frac{f(r^i)}{r^i} \left[1 - \cos\left(\mathbf{k} \bullet \mathbf{r}^i\right)\right] \right\}$$

$$V \equiv \sum_i m_i \left\{ \frac{\left(r^i \dfrac{\partial f(r^i)}{\partial r^i} - f(r^i) \right)}{r^i} \left[\frac{1}{2} \left(\frac{\mathbf{k} \bullet \mathbf{r}^i}{r^i} \right)^2 + \frac{\cos(\mathbf{k} \bullet \mathbf{r}^i)}{(r^i)^2} \right] \right\} \tag{22.4}$$

Again, recall that the equilibrium loci $\mathbf{r}^i$ are taken to cover the position vector $\mathbf{r}$.

In general, Cauchy's equations always yield three distinct speeds of propagation: two for mutually orthogonal displacements in the surface of a plane wave, and one for a displacement normal to the plane. The latter, he argued, is invisible to the eye, while the former two speeds, which correspond to optical waves, must reduce to one in an isotropic medium. Cauchy had considerable difficulty reducing the system to isotropy and spent a great deal of effort demonstrating that the conditions at which he arrived satisfy the appropriate requirements. He eventually found that under isotropy the equation of motion (22.3) expressed in terms of U, V becomes:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = -\left(U + \frac{1}{k}\frac{\partial V}{\partial k} \right)\mathbf{u} - (\mathbf{k} \bullet \mathbf{u}) \frac{\partial \left(\dfrac{1}{k}\dfrac{\partial V}{\partial k} \right)}{\partial k} \mathbf{k}$$

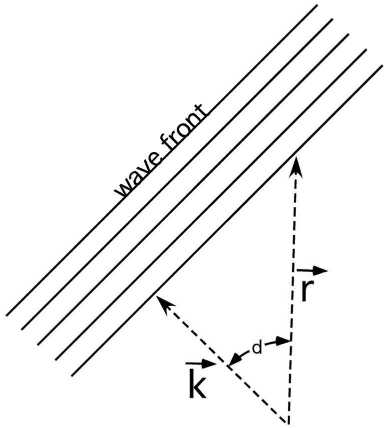
If the invisible third wave is ignored, so that the disturbance may be supposed to occur entirely in the plane of the wavefront, then $\mathbf{k} \bullet \mathbf{u}$ vanishes, and the equation transforms into one that has the same form as that of an harmonic oscillator[31]:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = -s^2 \mathbf{u} \text{ where } s^2 = U + \frac{1}{k}\frac{\partial V}{\partial k} \tag{22.5}$$

The two equations (22.4) and (22.5) entail a dispersion formula because the wave speed is just s/k, and k, which is inversely proportional to the wavelength, appears in s via U, V. Note that the speed of propagation apparently incorporates the angle d between the vector $\mathbf{k}$ that is normal to the front and the vector directed to the point $\mathbf{r}$ at which the disturbance is evaluated. That angle continues to appear in Cauchy's final dispersion formula, but it merely represents the *apparent* speed that would be measured when looking in any direction other than the one in which the wave is propagating, as in the diagram below. In his final expression for U, V, $\mathbf{k} \bullet \mathbf{r}$ is written $rk\cos(d)$, with $k\cos(d)$ accordingly representing the projection of the propagation vector $\mathbf{k}$ in the direction $\mathbf{r}$. Since

[31] Propagation, and not simply oscillation, occurs because s^2, the analog of the harmonic coefficient, is a function of $\mathbf{k}$ and $\mathbf{r}$. Cauchy developed the solution in detail.

dispersion relations – and, in particular, the measured values for refractive indices as a function of wavelength – concern the speed in the direction normal to the front, cos(d) may be set to one for comparison with experiment. Cauchy retained the angle for generality, but this does not matter in the end, as we will now see, because he replaced all of the lattice-dependent terms with constants.



To obtain a dispersion relation, Cauchy, in a crucial step, turned his expressions for U and $\partial V/\partial k$ into series by expanding the term $\cos(\mathbf{k} \bullet \mathbf{r}^i)$ in powers of k. In virtue of (22.5) the following series for Ω^2, the square of the wave speed, results:

$$\Omega^2 = \frac{s^2}{k^2} = \frac{\left[U + \frac{1}{k}\frac{\partial V}{\partial k}\right]}{k^2} = \sum_{j=1}\left[k^{(2j-2)}\right]a_j$$

where (22.6)

$$a_j \equiv \left\{\left(\frac{(-1)^{(j-1)}}{(2j)!}\right)\left[\sum_i m_i (r^i)^{(2j-1)}(\cos(d))^{2j}\left(f(r^i) + \left(\frac{(\cos(d))^2}{2j+1}\right)\left[r^i\frac{\partial f(r^i)}{\partial r^i} - f(r^i)\right]\right)\right]\right\}$$

Cauchy simply took the a_j to be constants that pertain to a particular medium, which accordingly gave him an expression for the wave speed as

$$\Omega = \sqrt{a_1 + a_2k^2 + a_3k^4 + \ldots} \qquad (22.7)$$

Since k is just $2\pi/\lambda$, where λ is the wavelength, Cauchy's equation is precisely the same in form as the one that Fresnel had obtained for the wave speed in his unpublished manuscript of July 1822, namely $\sqrt{n}\sqrt{P - \frac{Q}{\lambda^2} + \frac{R}{\lambda^4} - \&c.}$ (note that Fresnel included the alternating signs that Cauchy incorporated into his a_j constants). Cauchy's analysis is certainly vastly more intricate and mathematically meticulous, as was his wont, than Fresnel's few pages from 1822.

Nevertheless, Fresnel had reasoned his way to the very same dispersion formula that Cauchy published in 1835 as a rigorous consequence of his elaborate theory.

We know that Cauchy had met with Fresnel to discuss matters of optics around the time that Fresnel wrote down his dispersion formula. And it is highly probable that he had not begun to work on equations for a system of particles until after that meeting. Indeed, his first work on a particle model did not appear until the addendum of 1827 in which he mentioned that Fresnel had arrived at similar results. Turn now to Fresnel's brief, unpublished deduction of July, 1822 in order to expose his principal conceptions and thereby to compare them with the ones that underpin Cauchy's intricate analysis.

What is most interesting about Fresnel's route to dispersion is that he began directly with a propagating wave and considered the forces involved in its motion. In his figure (see below), a wave of form $\sin 2\pi(x/\lambda)$ displaces the ether's particles. Fresnel then examined the effect on a particular "slice" of the wave-bearing medium that is exerted by two neighboring ones. His lines *mp*, *MP*, and $m'p'$ each represents a region of the ether displaced by the wave, with the regions spaced a distance *h* apart. He identified three factors that determine the forces which the neighboring slices exert on *MP*: first, the difference between their displacements, second their distance apart, and third, the "energy of the elasticity." These first two factors immediately yield expressions for the "action" (i.e. force) on *MP* by taking the differences $MP\text{-}m'p'$ and *MP*-*mp*.

If *P* is located at *x*, then p' is at $x{+}h$, and so the difference in displacements is proportional to $\sin(2\pi x/\lambda) - \sin(2\pi(x+h)/\lambda)$. A similar expression holds for the effect of slice *mp*, replacing *h* with $-h$. Fresnel next expanded the differences in the inverse powers of the wavelength to obtain two series which he then added to obtain the combined "action" on the slice *MP*.[32] At this point we already have, in effect, an appropriate dispersion series in the form $Ca\left[1 - \frac{h^2}{3\cdot 4}\frac{\varepsilon^2}{\lambda^2} + \frac{h^4}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^4}{\lambda^4} - \&c.\right]\frac{h^2}{2}\frac{\varepsilon^2}{\lambda^2}\sin\frac{\varepsilon x}{\lambda}$. Since only *h* and a proportionality constant change for the other slices, Fresnel concluded that the series retains the same form. The factor that multiplies the sine term in his expression, Fresnel asserted, expresses the "energy" of the force for a unit displacement. And, he continued, the period of the oscillation is inversely proportional to the square root of this factor – obviously considering the behavior of a particle through which the wave passes to obey the same rules as an harmonic oscillator whose coefficient is given by Fresnel's factor. The rest follows quite directly, and Fresnel concluded with an admittedly failed attempt to derive a general result that could carry a dispersion relation from one medium to another.

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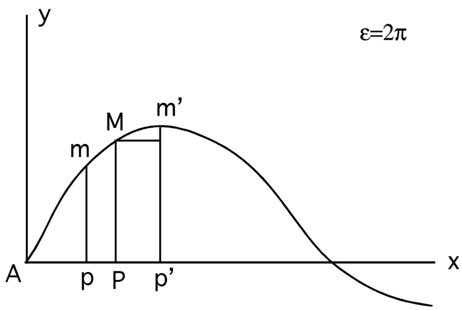
[32] He evidently did so by first rewriting $\sin(2\pi(x+h)/\lambda)$ as $\sin(2\pi x/\lambda)\cos(2\pi h/\lambda) - \cos(2\pi x/\lambda)\sin(2\pi h/\lambda)$ and then expanding the terms containing h/λ into series. He did the same for the effect of slice *mp*.

Fresnel's excursion is vastly less rigorous and detailed than Cauchy's, and yet it yields effectively the same result and, moreover, shares with it an interesting effort to reduce the system to the case of an harmonic oscillator – as Cauchy, after lengthy deductions, did in reaching his equation (22.5) above. Also like Fresnel, Cauchy expanded the factors in his equations that determine the forces involved into a series in the inverse powers of the wavelength. Further, Cauchy's entire structure depended directly on correcting the major *lacuna* in Fresnel's deduction of his surface of elasticity, namely Fresnel's assumption that only the particle in question could be displaced, holding all the others fixed in situ. That assumption led to the linear transformation from which the elasticity surface emerged. Cauchy fixed the *lacuna*.

All of this is not likely to have been coincidental. It seems probable that Fresnel showed Cauchy his notes in the summer of 1822 or thereabouts, and that Cauchy took from those notes the essential idea to express the actions of the particles in terms of a series in the wavelengths, and to do so by generating a related equation of motion in harmonic form, one in which the coefficient contained the requisite wavelength series. Yet Cauchy never mentioned, in print at least, anything about Fresnel's work on dispersion. If the eponymous title of a formula should accrue to its first producer, or at least to the one who first developed the elements, subsequently elaborated, of a foundation for it, then perhaps "Cauchy's dispersion formula" should be reassigned to Fresnel, not least because Cauchy may have seen Fresnel's work. [*Bibliothèque de l'Institut*, MS 3411, pp. 64–7. July, 1822. Unpublished manuscript by Fresnel on dispersion]

Essais théoriques sur la dispersion

La courbe des déplacemens moleculaires est toujours sinusoidale: elle le sera donc à tous les instans dans une [?]mération résultant des détour d'une onde sur elle-même. Soit $y = \sin(x/\lambda)\varepsilon$ L'équation de cette courbe à une certain instant;



Il s'agit de détermine l'action exercée sur la tranche du milieu vibrant correspondant à l'ordonée MP par deux tranches équidistantes *mp* et $m'p'$. Je représente Pp et Pp' par h. L'action exercée sur la tranche en MP par la tranche en $m'p'$ est toutes choses egales d'ailleurs proportionelle au déplacement relatif $m'p' - MP$: elle dépend en outre de la distance h et de l'énergie de l'élasticité; elle est donc égale à un coëfficient constant dépendant de ces quantités, multiplié par $m'p' - MP$.

$$C(MP - m'p') = Ca\left[-h\tfrac{\varepsilon}{\lambda}\cos\tfrac{\varepsilon x}{\lambda} + \tfrac{h^2}{2}\tfrac{\varepsilon^2}{\lambda^2}\sin\tfrac{\varepsilon x}{\lambda} + \tfrac{h^3}{2\cdot 3}\tfrac{\varepsilon^3}{\lambda^3}\cos\tfrac{\varepsilon x}{\lambda} - \tfrac{h^4}{2\cdot 3\cdot 4}\tfrac{\varepsilon^4}{\lambda^4}\sin\tfrac{\varepsilon x}{\lambda} - \&c.\right]$$
$$C(MP - mp) = Ca\left[h\tfrac{\varepsilon}{\lambda}\cos\tfrac{\varepsilon x}{\lambda} + \tfrac{h^2}{2}\tfrac{\varepsilon^2}{\lambda^2}\sin\tfrac{\varepsilon x}{\lambda} - \tfrac{h^3}{2\cdot 3}\tfrac{\varepsilon^3}{\lambda^3}\cos\tfrac{\varepsilon x}{\lambda} - \tfrac{h^4}{2\cdot 3\cdot 4}\tfrac{\varepsilon^4}{\lambda^4}\sin\tfrac{\varepsilon x}{\lambda} + \&c.\right]$$

Adjoutant ces deux actions des deux tranches équidistantes:

$$Ca\left[\frac{h^2}{2}\frac{\varepsilon^2}{\lambda^2}\sin\frac{\varepsilon x}{\lambda} - \frac{h^4}{2\cdot 3\cdot 4}\frac{\varepsilon^4}{\lambda^4}\sin\frac{\varepsilon x}{\lambda} + \frac{h^6}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^6}{\lambda^6}\sin\frac{\varepsilon x}{\lambda} - \&c.\right]$$
$$ou,$$
$$Ca\left[1 - \frac{h^2}{3\cdot 4}\frac{\varepsilon^2}{\lambda^2} + \frac{h^4}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^4}{\lambda^4} - \&c.\right]\frac{h^2}{2}\frac{\varepsilon^2}{\lambda^2}\sin\frac{\varepsilon x}{\lambda}$$

On voit que cette force accélératrice, pour les mêmes valeurs de h et de C est toujours proportionelle à $a\sin\frac{\varepsilon x}{\lambda}$, c'est à dire à l'espace à parcourir par la molécule M pour arriver à l'axe AX; ainsi toutes les molécules y arrivent en même tems, et à chaqueinstant de son oscillation la courbe se trouve toujours sinusoïdales, lors même que l'action moléculaire s'étend à des distances sensibles relativement à λ. Notre calcul suppose seulement que la série est convergente, c'est à dire que $\frac{\varepsilon h}{\lambda}$ est plus petit que l'unité, ou h moindre que le tiers de λ.

L'expression de la force exercée par tous les autres couples de tranches équidistantes aurait la même forme; il n'y aurait que h et C qui changerait de valeur:

$$Ch^2\left[1 - \frac{h^2}{3\cdot 4}\frac{\varepsilon^2}{\lambda^2} + \frac{h^4}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^4}{\lambda^4} - \&c.\right]a\frac{\varepsilon^2}{\lambda^2}\sin\frac{\varepsilon x}{\lambda};$$
$$C'h'^2\left[1 - \frac{h'^2}{3\cdot 4}\frac{\varepsilon^2}{\lambda^2} + \frac{h'^4}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^4}{\lambda^4} - \&c.\right]a\frac{\varepsilon^2}{\lambda^2}\sin\frac{\varepsilon x}{\lambda}$$

Faisant la somme de toutes ces actions, on a:

$$\Big[Ch^2 + C'h'^2 + \&c - \left(Ch^4 + C'h'^4 + \&c\right)\frac{\varepsilon^2}{3\cdot 4\cdot \lambda^2}$$
$$+\left(Ch^6 + C'h'^6 + \&c\right)\frac{\varepsilon^4}{3\cdot 4\cdot 5\cdot 6\cdot \lambda^4} - \&c\Big]a\frac{\varepsilon^2}{\lambda^2}\sin\frac{\varepsilon x}{\lambda};$$

l'écartement $MP = a\sin\frac{\varepsilon x}{\lambda}$.

Par conséquent le facteur constant qui exprime l'énergie de la force pour un écartement égal à 1 est l'expression ci-dessus dans laquelle on aurait supprimé le facteur $a \sin \frac{\varepsilon x}{\lambda}$. Mais la durée de l'oscillation est en raison inverse de la racine carrée de ce coefficient et par conséquent en raison inverse de

$$\frac{\varepsilon}{\lambda}\sqrt{Ch^2+C'h'^2+\&c-(Ch^4+C'h'^4+\&c)\frac{1}{3\cdot 4}\frac{\varepsilon^2}{\lambda^2}+(Ch^6+C'h'^6+\&c)\frac{1}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^4}{\lambda^4}-\&c}$$

Pour une onde d'une longeur égale à λ', on aurait:

$$\frac{\varepsilon}{\lambda'}\sqrt{Ch^2+C'h'^2+\&c-(Ch^4+C'h'^4+\&c)\frac{1}{3\cdot 4}\frac{\varepsilon^2}{\lambda'^2}+(Ch^6+C'h'^6+\&c)\frac{1}{3\cdot 4\cdot 5\cdot 6}\frac{\varepsilon^4}{\lambda'^4}-\&c}$$

En faisant

$$Ch^2+C'h'^2+\&c = P;\quad \frac{(Ch^4+C'h'^4+\&c)\varepsilon^2}{3\cdot 4} = Q;\quad \frac{(Ch^6+C'h'^6+\&c)\{\varepsilon^4\}}{3\cdot 4\cdot 5\cdot 6} = R;$$

Le 1er expression devient, $\frac{\varepsilon}{\lambda}\sqrt{P-\frac{Q}{\lambda^2}+\frac{R}{\lambda^4}-\&c.}$

Et le second $\frac{\varepsilon}{\lambda'}\sqrt{P-\frac{Q}{\lambda'^2}+\frac{R}{\lambda'^4}-\&c.}$

Mais les vitesses de propagation sont en raison inverse des durées d'oscillation; elles sont proportionelles pour le même milieu aux deux expressions,

$$\frac{\varepsilon}{\lambda}\sqrt{P-\frac{Q}{\lambda^2}+\frac{R}{\lambda^4}-\&c.}\,,\ \text{et},\ \frac{\varepsilon}{\lambda'}\sqrt{P-\frac{Q}{\lambda'^2}+\frac{R}{\lambda'^4}-\&c.}$$

Les quantités P,Q,R sont des fonctions des intervalles h, h', h'''& c. des coëfficiens correspondant C, C', C''& $c\ldots$, ou, en d'autres termes, des intégrales dans les diff.lles h est la variable et C une fonction de h qui diminue rapidement à mesure que h augmente, ces intégrales étant prises jusqu'à $h=\infty$.

$$Ch^2 + C'h'^2 + \&c = P = \int_{-\infty}^{+\infty} h^2\varphi(h)dh\ldots;$$

$$Ch^4 + C'h'^4 + \&c = \int_{-\infty}^{+\infty} h^4\varphi(h)dh = \frac{3\cdot 4\cdot Q}{\varepsilon^2}$$

$$Ch^{\{6\}} + C'h'^6 + \&c = \int_{-\infty}^{+\infty} h^6\varphi(h)dh = \frac{3\cdot 4\cdot 5\cdot 6}{\varepsilon^4}\cdot R$$

$+\infty$ et $-\infty$ n'indique pas ici des quantités infinies ni mêmes grandes relativement à λ, puisqu'alors nos séries fondamentales n'étant plus convergents deviendrait des expressions illusoires: $+\infty$ et $-\infty$ indiquent seulement des limites de la sphère d'activité sensible de l'action réciproque des tranches du milieu vibrant.

Si sans fixer la loi suivant laquelle cette force décroit avec la distance, on supposait que la loi est semblable dans tous les milieux, c'est à dire que le fonction φ reste la même à facteur près, pour un autre milieux, des quantités P', Q', R' seraient égales à nP, nQ, nR, et les deux vitesses de propagation correspond[tes] aux long d'ondulat. λ et λ' seraient:

$$\sqrt{nP-\frac{nQ}{\lambda^2}+\frac{nR}{\lambda^4}-\&c.}\ ,\ \text{et,}\ \sqrt{nP-\frac{nQ}{\lambda'^2}+\frac{nR}{\lambda'^4}-\&c.};\text{ou,}$$

$$\sqrt{n}\sqrt{P-\frac{Q}{\lambda^2}+\frac{R}{\lambda^4}-\&c.}\ ,\ \text{et,}\ \sqrt{n}\sqrt{P-\frac{Q}{\lambda'^2}+\frac{R}{\lambda'^4}-\&c.}$$

et par conséquent la dispersion serait la même dans les milieux également réfringens, ce qui est contraire à l'experience. On ne peut donc pas supposer que les coefficiens P', Q', R' soient des coefficiens P, Q, R multipliées par le même fonction n,

L'expression générale du rapport de réfraction pour le passage des ondes λ de l'air d'un milieu réfringent, est, en appelant v la vitesse de λ dans l'air, $\frac{v}{\sqrt{P-\frac{Q}{\lambda^2}+\frac{R}{\lambda^4}-\&c.}}$: λ est la longeur d'ondulat[n] dans le milieu réfringent

ou, $\frac{v}{n\sqrt{1-\frac{A}{\lambda^2}+\frac{B}{\lambda^4}-\&c.}}$ pour le même milieu $\frac{v'}{n\sqrt{1-\frac{A}{\lambda'^2}+\frac{B}{\lambda'^4}-\&c.}}$, et une autre longeur d'onde $\frac{v''}{n\sqrt{1-\frac{A}{\lambda''^2}+\frac{B}{\lambda''^4}-\&c.}}$

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